Dynamical disentanglement approach to data analysis

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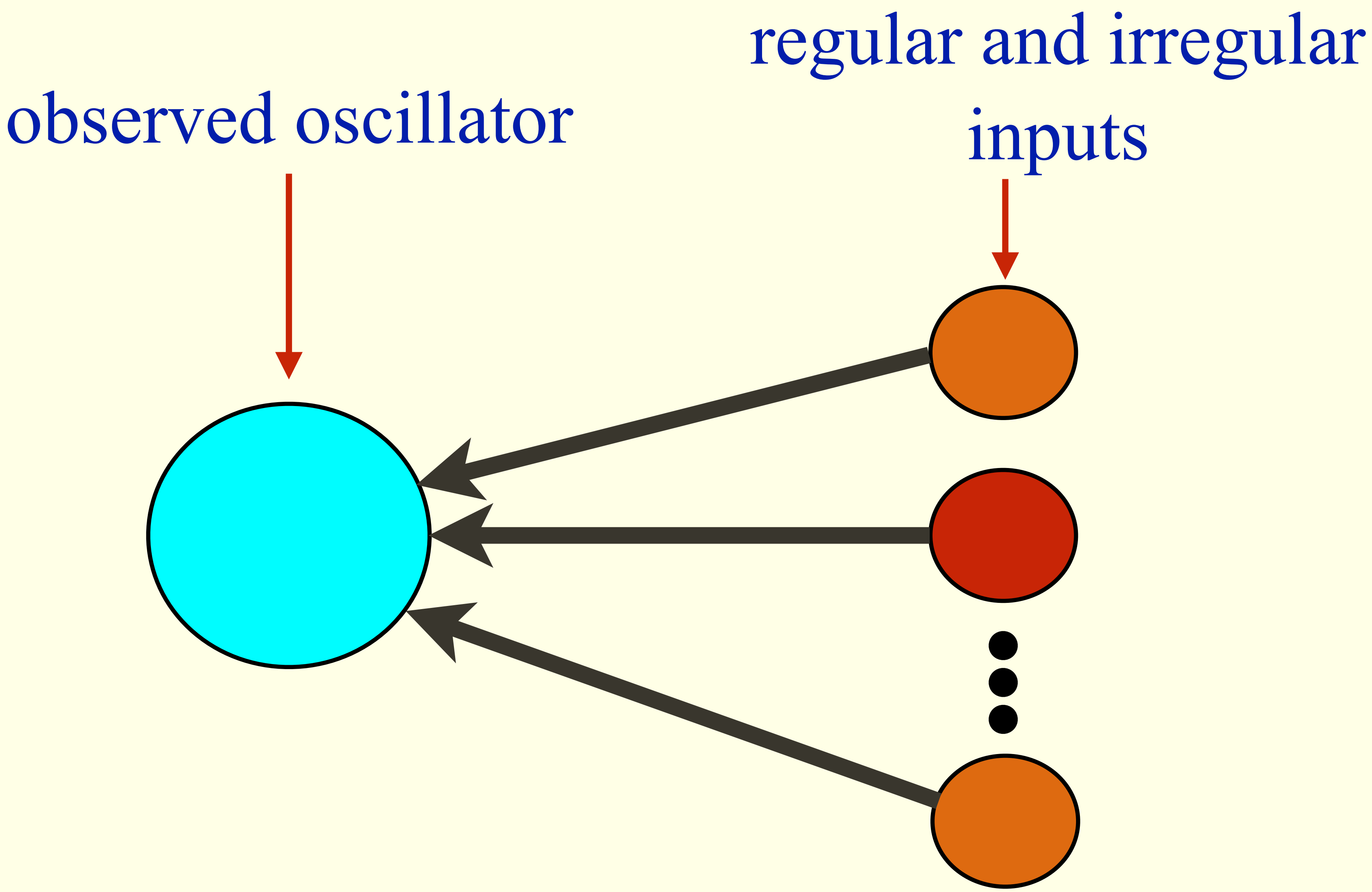
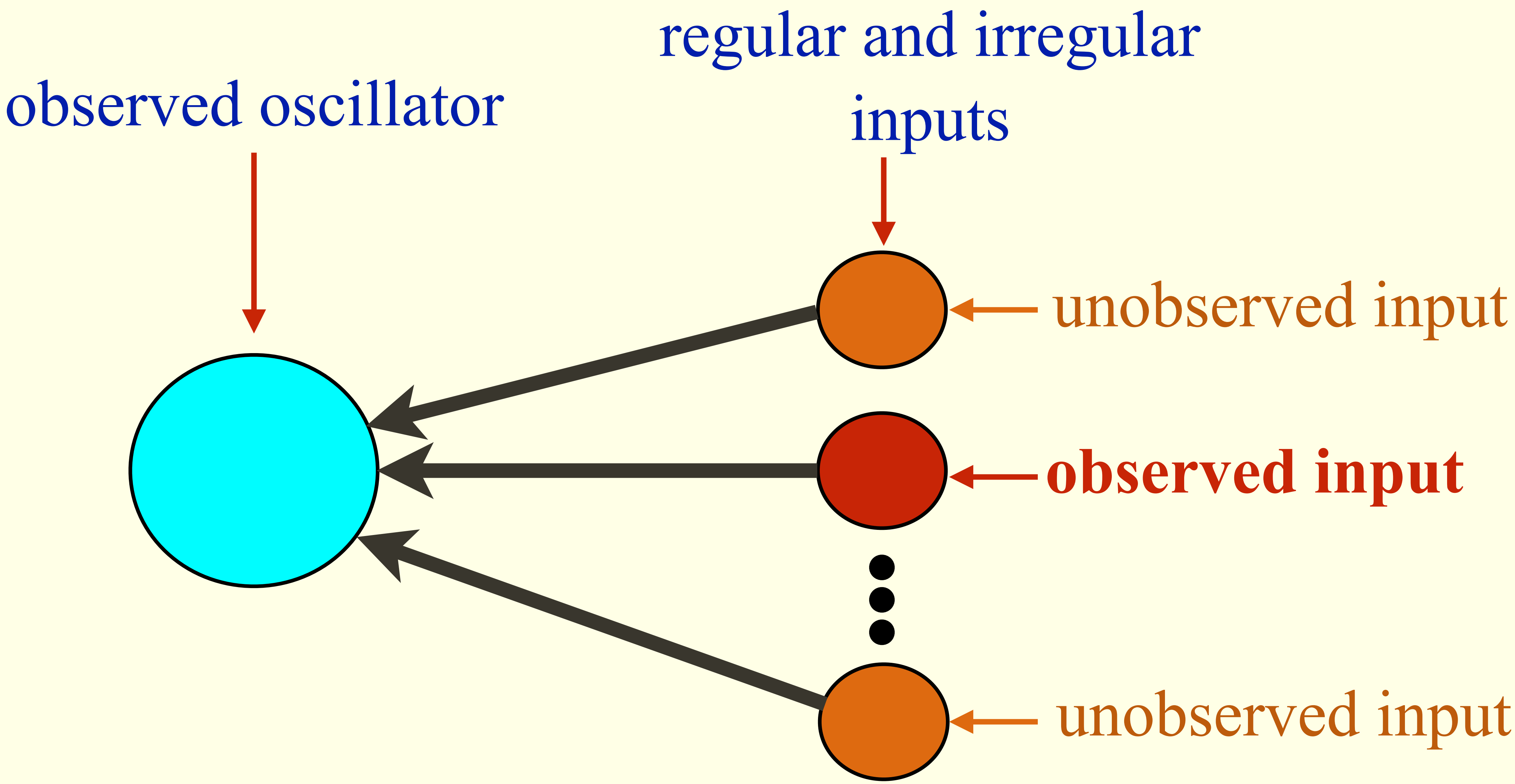
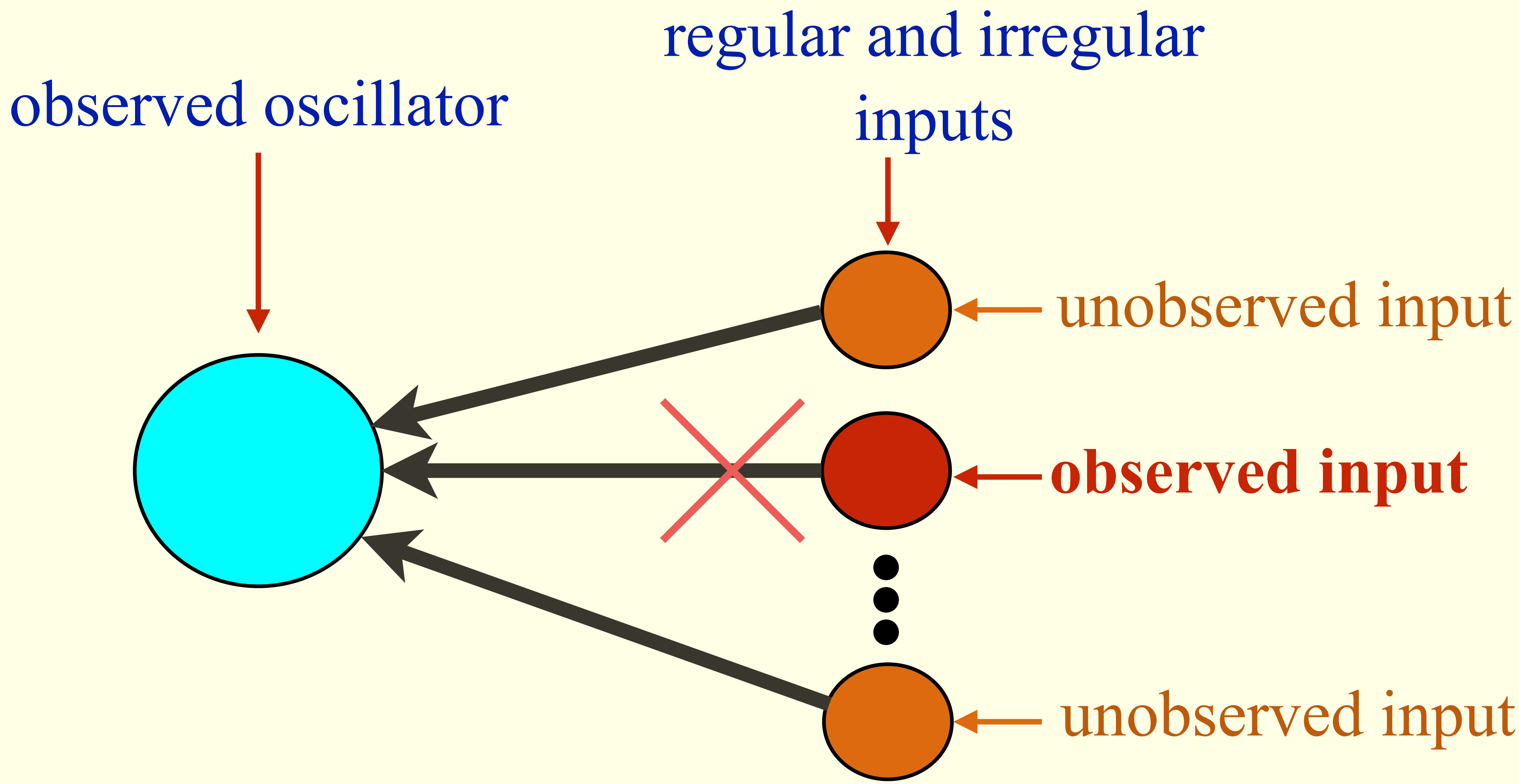
Dynamical disentanglement



Dynamical disentanglement

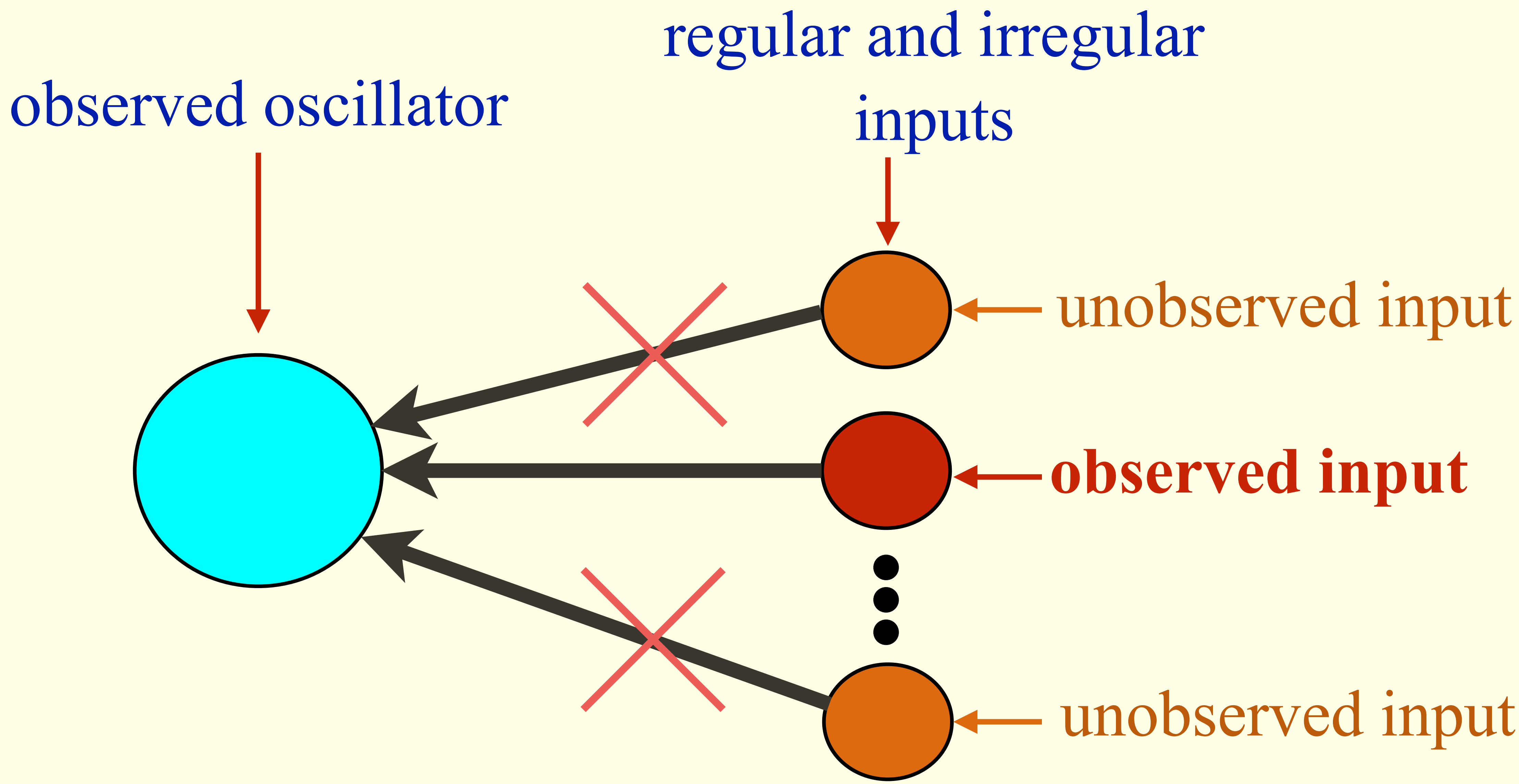


Dynamical disentanglement



Question 1: what would be dynamics of the oscillator if there were no observed input?

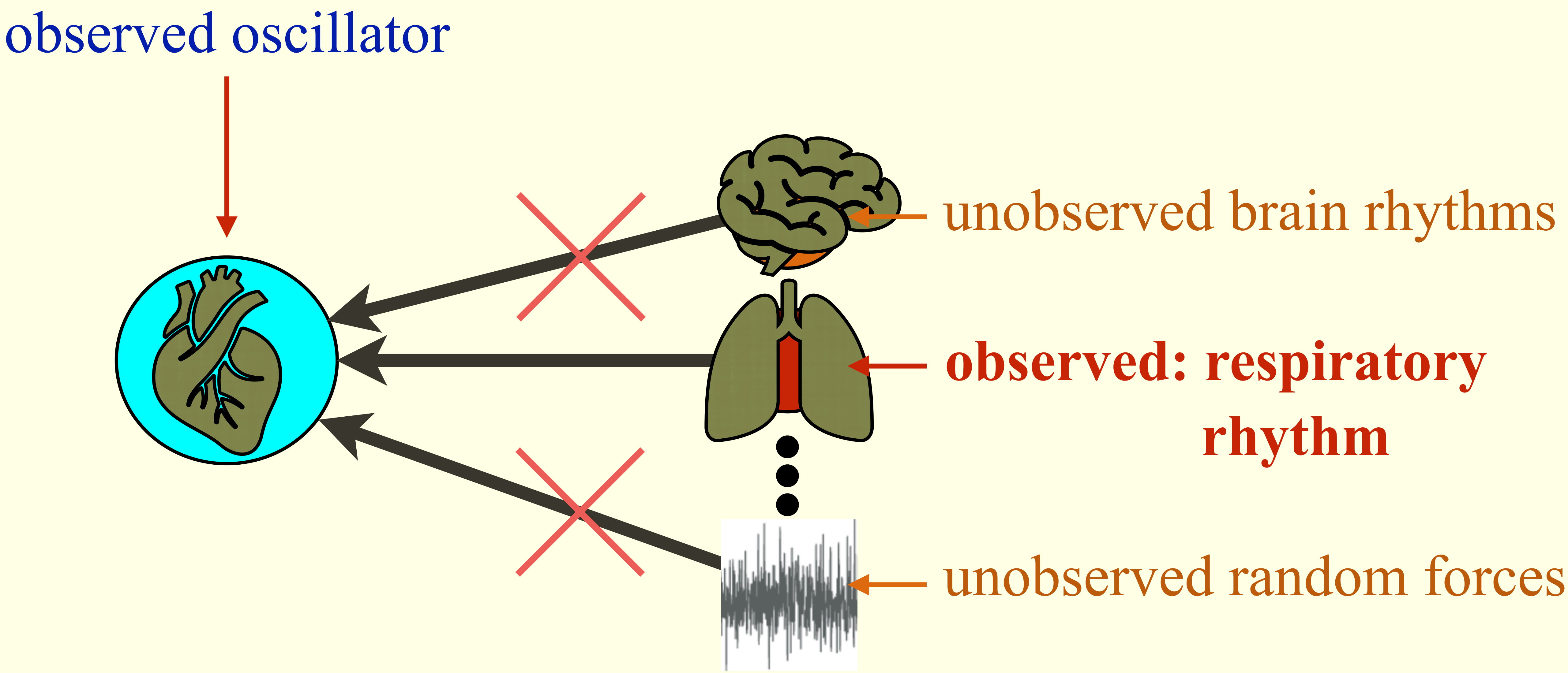
Dynamical disentanglement



Question 1: what would be dynamics of the oscillator if there were no observed input?

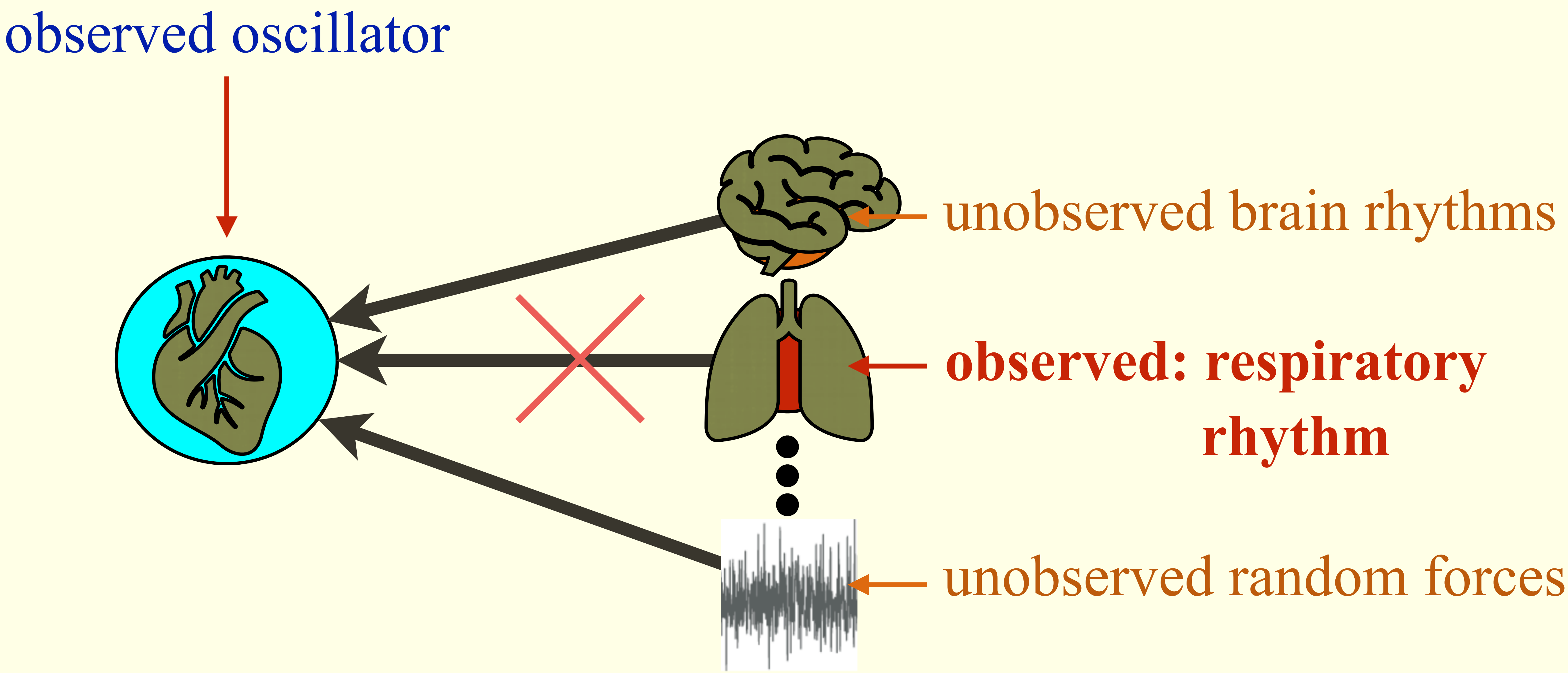
Question 2: what would be dynamics of the oscillator if there were no other inputs?

Dynamical disentanglement: an application



Question 1: how does the respiratory-related heart rate variability look like?

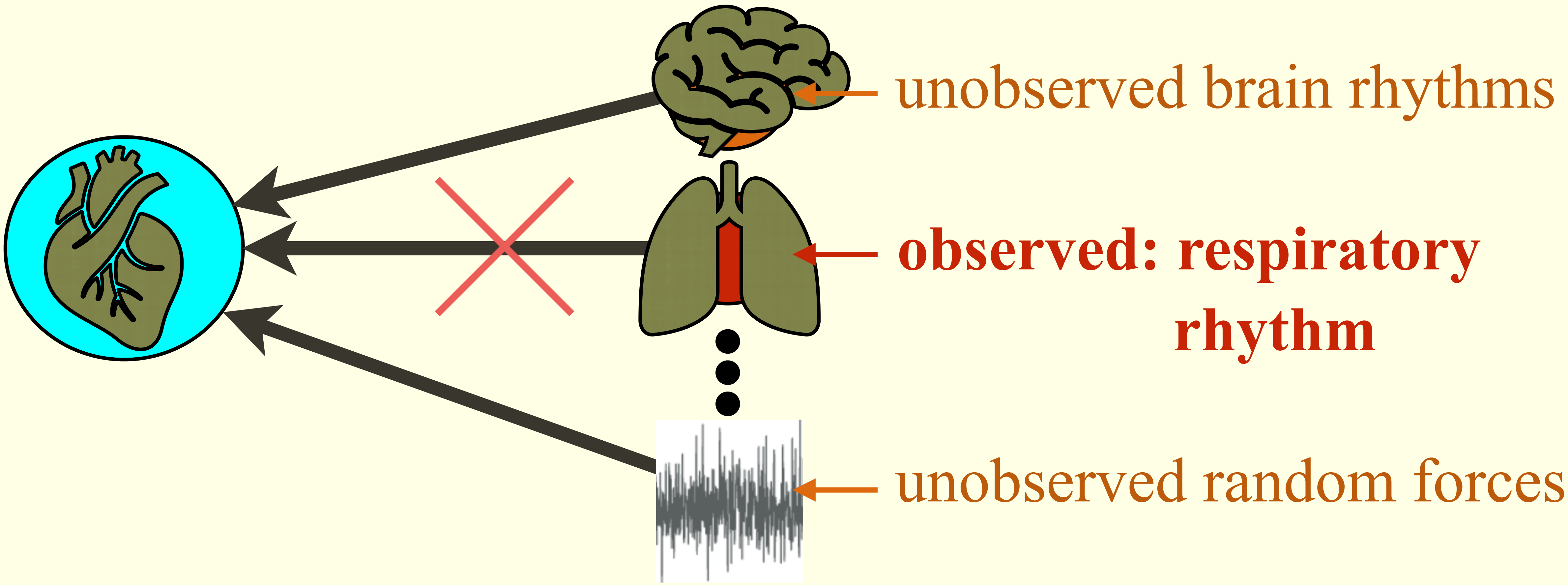
Dynamical disentanglement: an application



Question 1: how does the respiratory-related heart rate variability look like?

Question 2: what is the cardiac rhythm variability due to sources other than respiration?

Dynamical disentanglement: an application

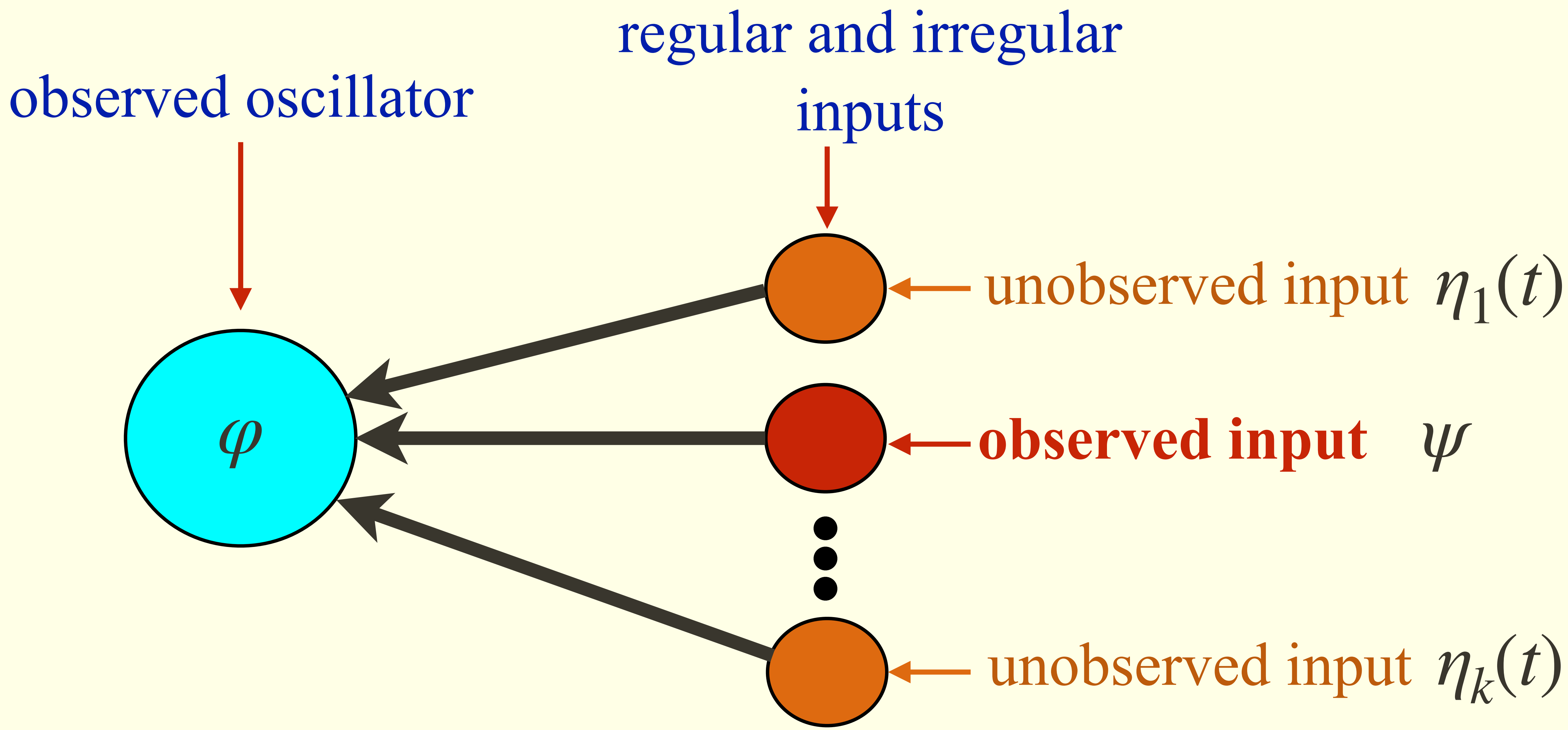


Question 1: how does the respiratory-related heart rate variability look like?

Question 2: what is the cardiac rhythm variability due to sources other than respiration?

Our approach is based on inference of the phase dynamics equation from observations

Dynamical disentanglement



Suppose we can reconstruct phase dynamics from observations:

$$\dot{\varphi} = \omega + Q(\varphi, \psi) + \sum_k Q_k(\varphi, \eta_k(t)) = \omega + Q(\varphi, \psi) + \zeta$$

Dynamical disentanglement

Suppose we can reconstruct phase dynamics from observations:

$$\dot{\varphi} = \omega + Q(\varphi, \psi) + \sum_k Q_k(\varphi, \eta_k(t)) = \omega + Q(\varphi, \psi) + \zeta$$

Question 1: what would be dynamics of the oscillator if there were no observed input?

→ we solve equation $\dot{\varphi} = \omega + \zeta$

Question 2: what would be dynamics of the oscillator if there were no other inputs?

→ we solve equation $\dot{\varphi} = \omega + Q(\varphi, \psi)$

Dynamical disentanglement: simple example

Suppose we have a noisy system, e.g.

$$\ddot{x} - 4(1 - \dot{x}^2)\dot{x} + x = p(t) = \varepsilon\cos(\nu t) + \zeta(t)$$

noise

For weak perturbation $p(t)$ the coupling function reads

$$\dot{\varphi} = \omega + Q(\varphi, \nu t) + Q_N(\varphi, \zeta(t))$$

Deterministic part $\dot{\varphi} = \omega + Q(\varphi, \nu t)$ describes the noise-free system

We use $\varphi, \psi = \nu t$ to infer Q via fit from observations

While fit $\approx$ averaging, the random perturbations are washed out

and we obtain equation $\dot{\varphi} \approx \omega + Q(\varphi, \nu t)$

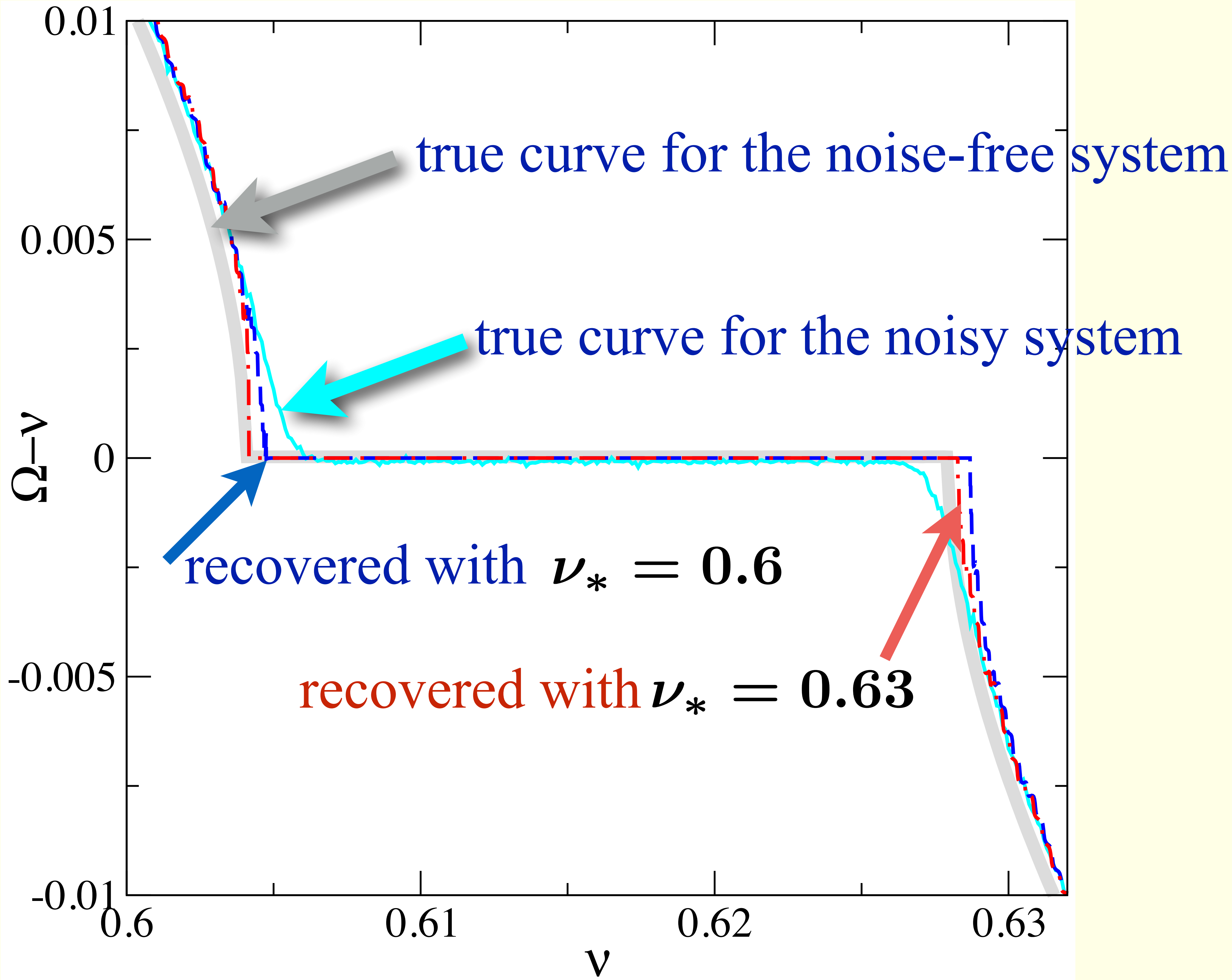
Dynamical disentanglement: simple example

- We obtain equation $\dot{\varphi} \approx \omega + Q(\varphi, \nu t)$ that describes noise-free system and we can solve it numerically for different ν to predict domain of locking

- Thus, we can find the Arnold tongue from a few measurements of noisy systems
  In experiments, phase can be estimated from data, e.g., with the help of the Hilbert Transform

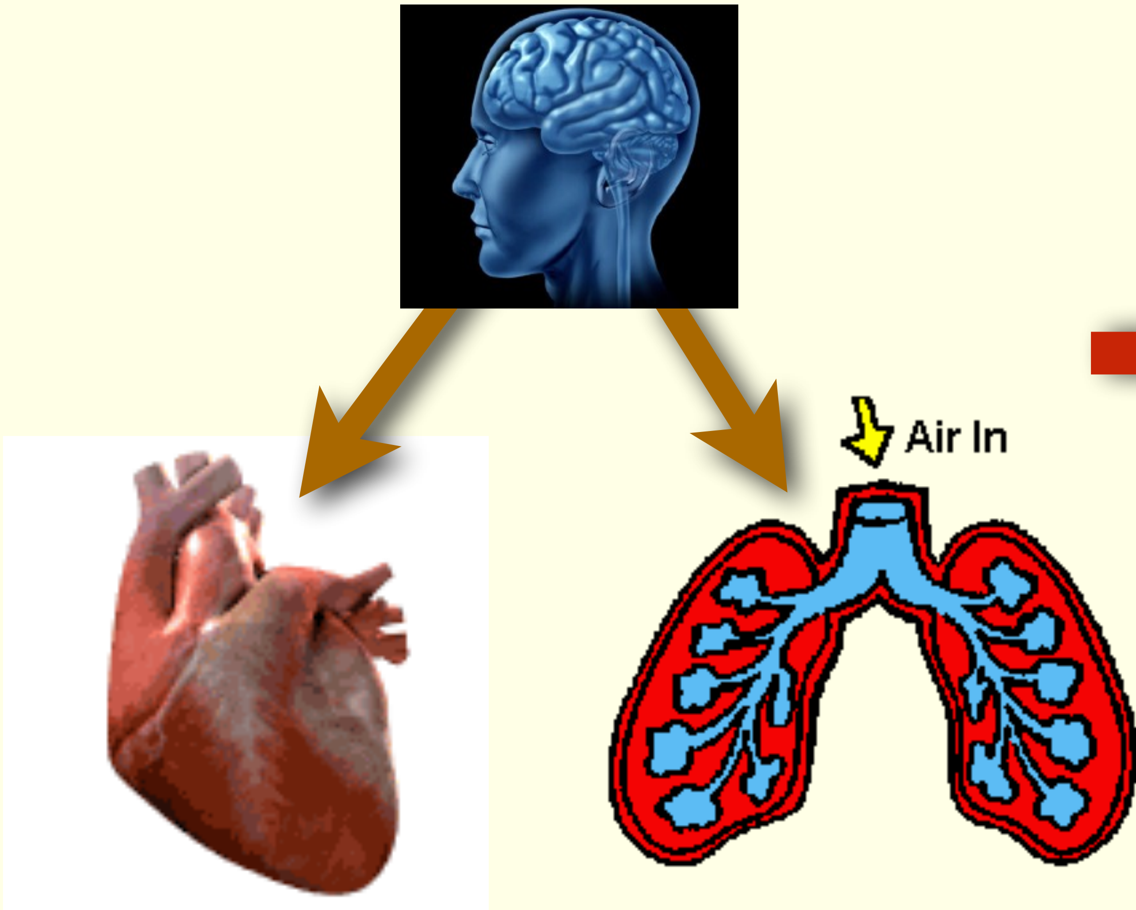
It works perfectly for weak noise and quite good for strong one!

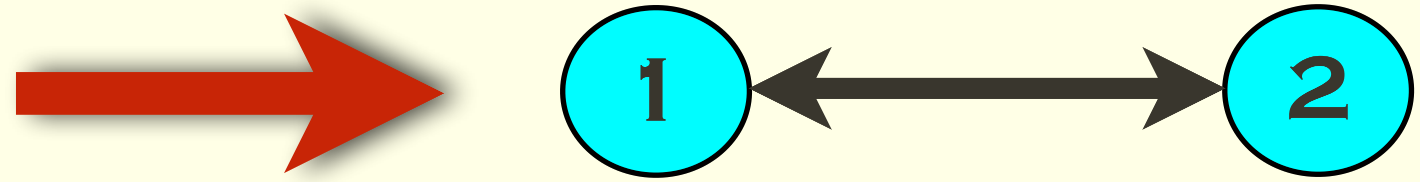
Noisy Rayleigh oscillator



$\varepsilon_* = 0.05$, $D = 0.05$

Main example: cardiorespiratory interaction





A model: two coupled self-sustained oscillators

Analysis: synchronization indices, directionality indices

reconstruction of the phase dynamics model

Our main interest: respiratory-related heart rate variability

Coupled oscillators: phase description

A model: two coupled self-sustained oscillators

$$\dot{\varphi}_1 = \omega_1 + Q_1(\varphi_1, \varphi_2)$$
$$\dot{\varphi}_2 = \omega_2 + Q_2(\varphi_1, \varphi_2)$$

Coupled oscillators: phase description

A model: two coupled self-sustained oscillators

$$\dot{\varphi}_1 = \omega_1 + Q_1(\varphi_1, \varphi_2)$$
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coupling functions

These equations can be reconstructed from data

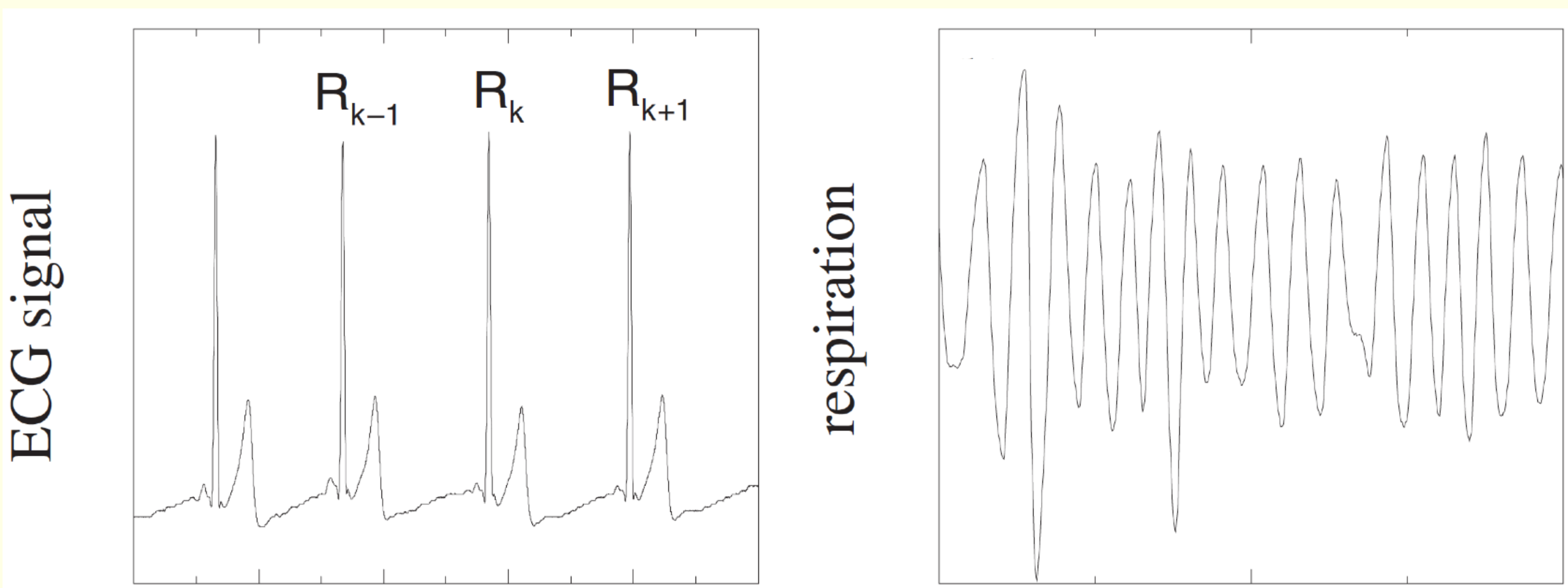
Cardiorespiratory interaction in adults

Björn Kralemann[1], Matthias Frühwirth[2], Arkady Pikovsky[3], Michael Rosenblum[3], Thomas Kenner[4], Jochen Schaefer[5] & Maximilian Moser[2,4]

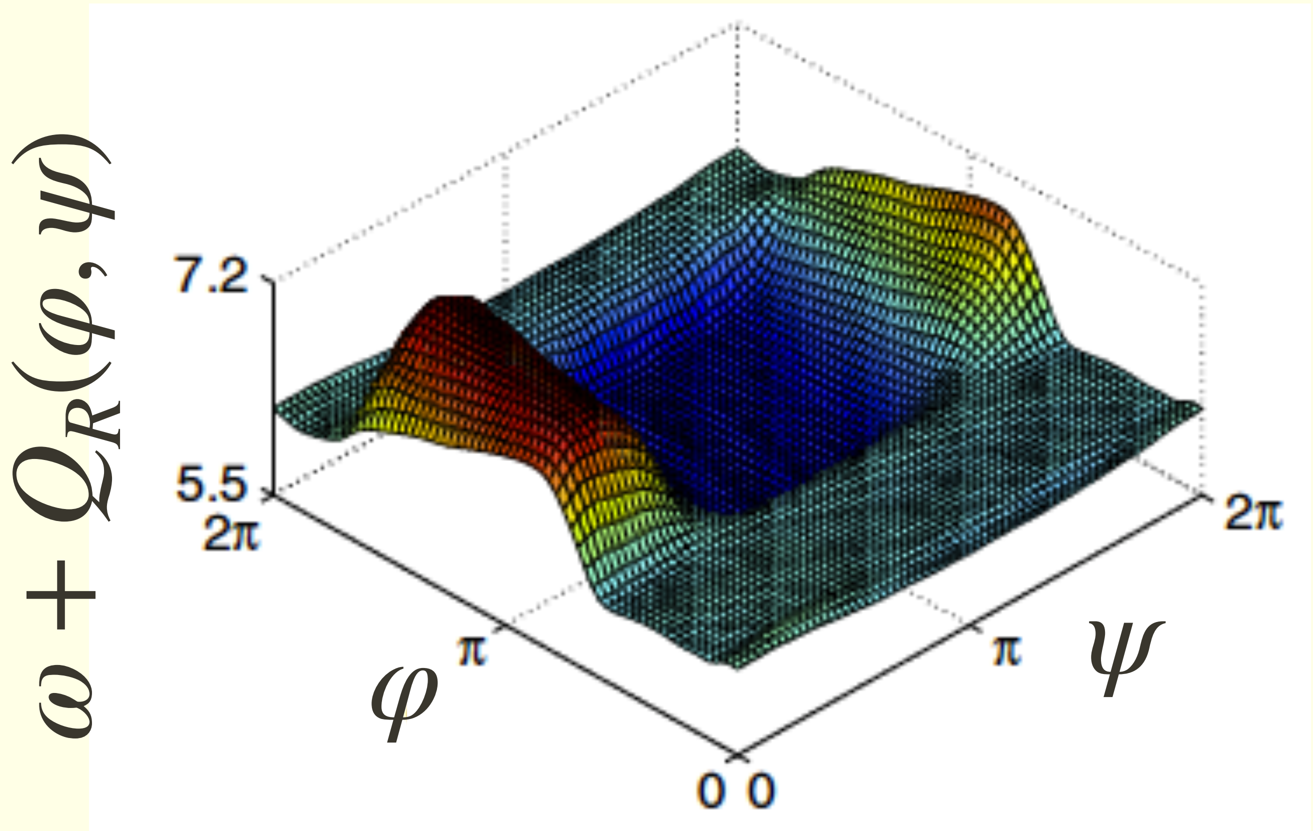
nature COMMUNICATIONS

Experiments on healthy humans

- Spontaneous respiration, supine position, rest state
- Data: ECG, arterial pulse, respiration



Cardiac dynamics: the coupling function



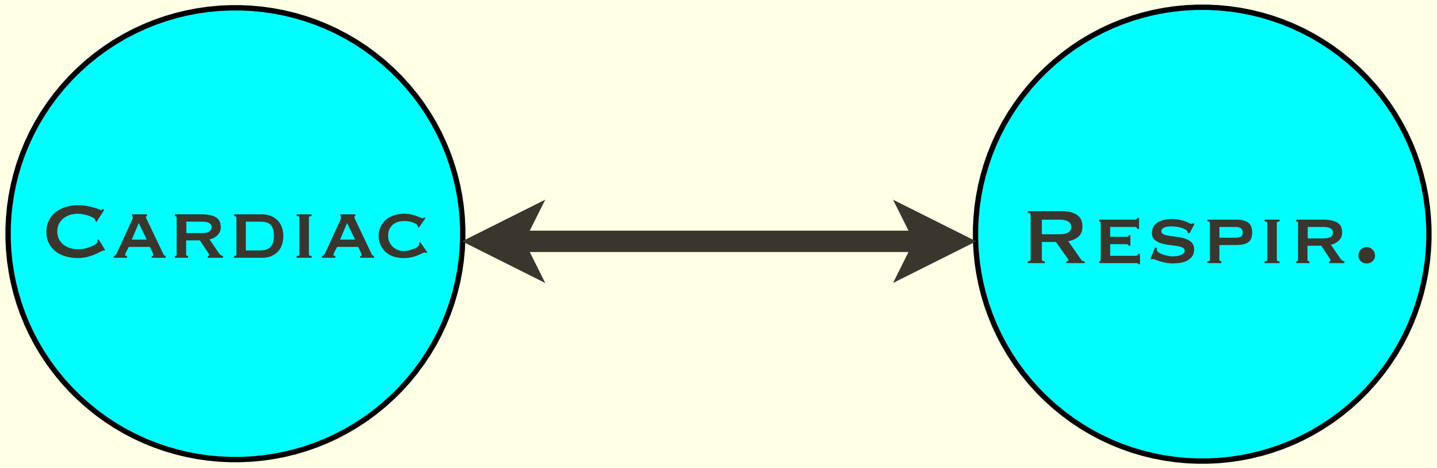
Cardiac phase from ECG

Phase of respiration

$$\dot{\varphi} = \omega + Q_R(\varphi, \psi)$$

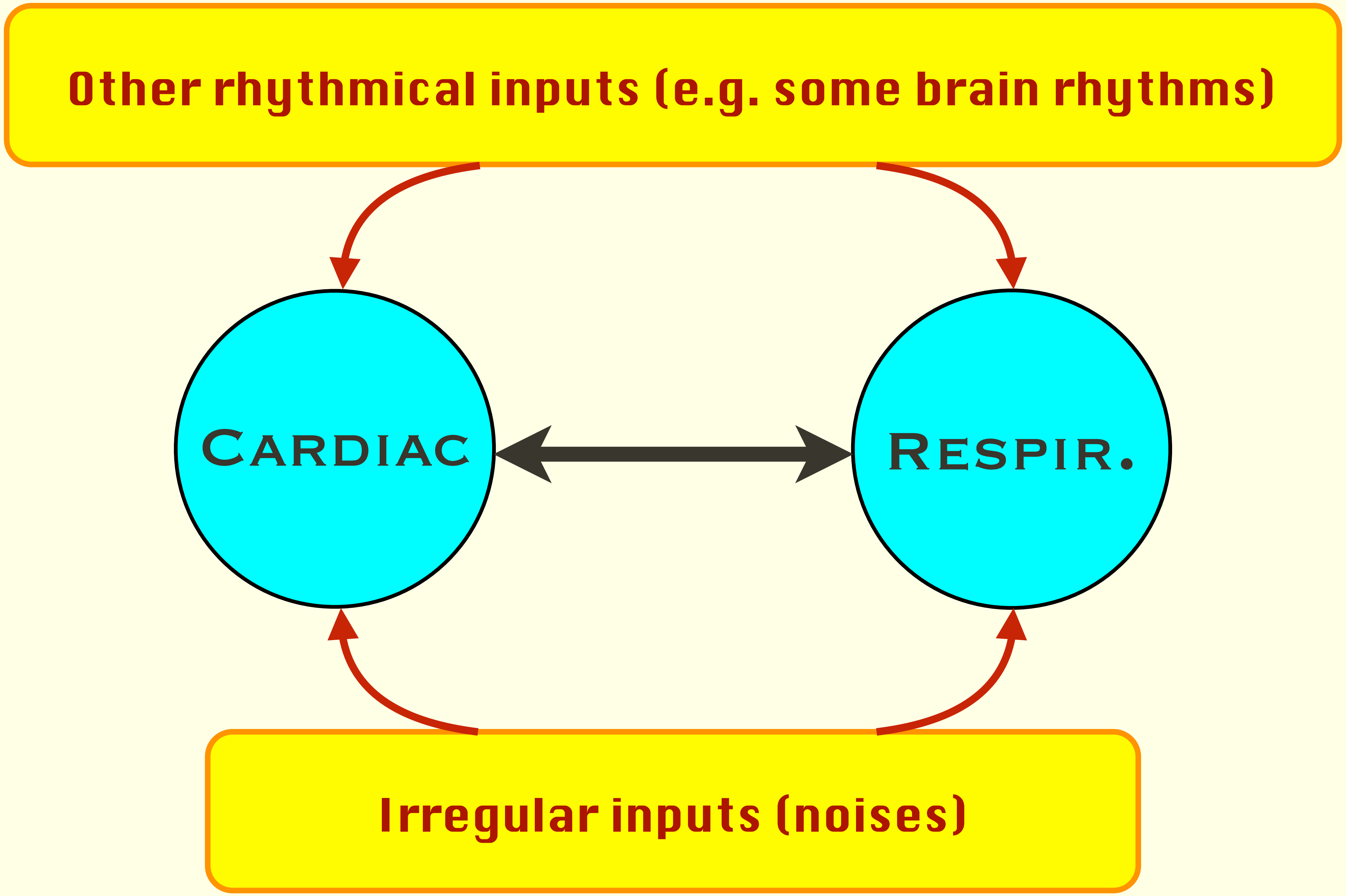
stands for 'respiration'

We used the model of two coupled oscillators…



…but it is too simplistic!

Interaction with the environment



Phase dynamics with account of many inputs

Cardiac phase from ECG

Phase of respiration

$$\dot{\varphi} = \omega + Q_R(\varphi, \psi) + \sum_s Q_s(\varphi, \eta_s) + \zeta$$

other inputs

intrinsic fluctuations

For weak inputs we expect to have a sum of coupling functions for different inputs, while for stronger inputs we expect cross-terms

Thus, we have two terms:

$Q_R(\varphi, \psi)$ describes variability due to respiration only

$\xi = \sum_s Q_s(\varphi, \eta_s) + \zeta$ describes effect of everything else except respiration

Phase dynamics with account of many inputs

Thus, we have two terms:

$Q_R(\varphi,\psi)$ describes variability due to respiration only

$\xi = \sum_s Q_s(\varphi,\eta_s) + \zeta$ describes effect of everything else except respiration

Hence, we achieve a decomposition:

$$\dot{\varphi} - \omega = Q_R(\varphi,\psi) + \xi$$

- $\dot{\varphi} - \omega$: Heart rate variability (HRV)
- $Q_R(\varphi,\psi)$: variability due to respiration (RSA-HRV)
- ξ: variability due to everything else (non-RSA-HRV)

RSA= respiratory sinus arrhythmia

Phase dynamics with account of many inputs

Hence, we achieve a decomposition:

$$\dot{\varphi} - \omega = Q_R(\varphi, \psi) + \xi$$

Heart rate variability (HRV)

variability due to respiration (RSA-HRV)

variability due to everything else (non-RSA-HRV)

Practically: we estimate Q_R from time series $\dot{\varphi}_k, \varphi_k, \psi_k$

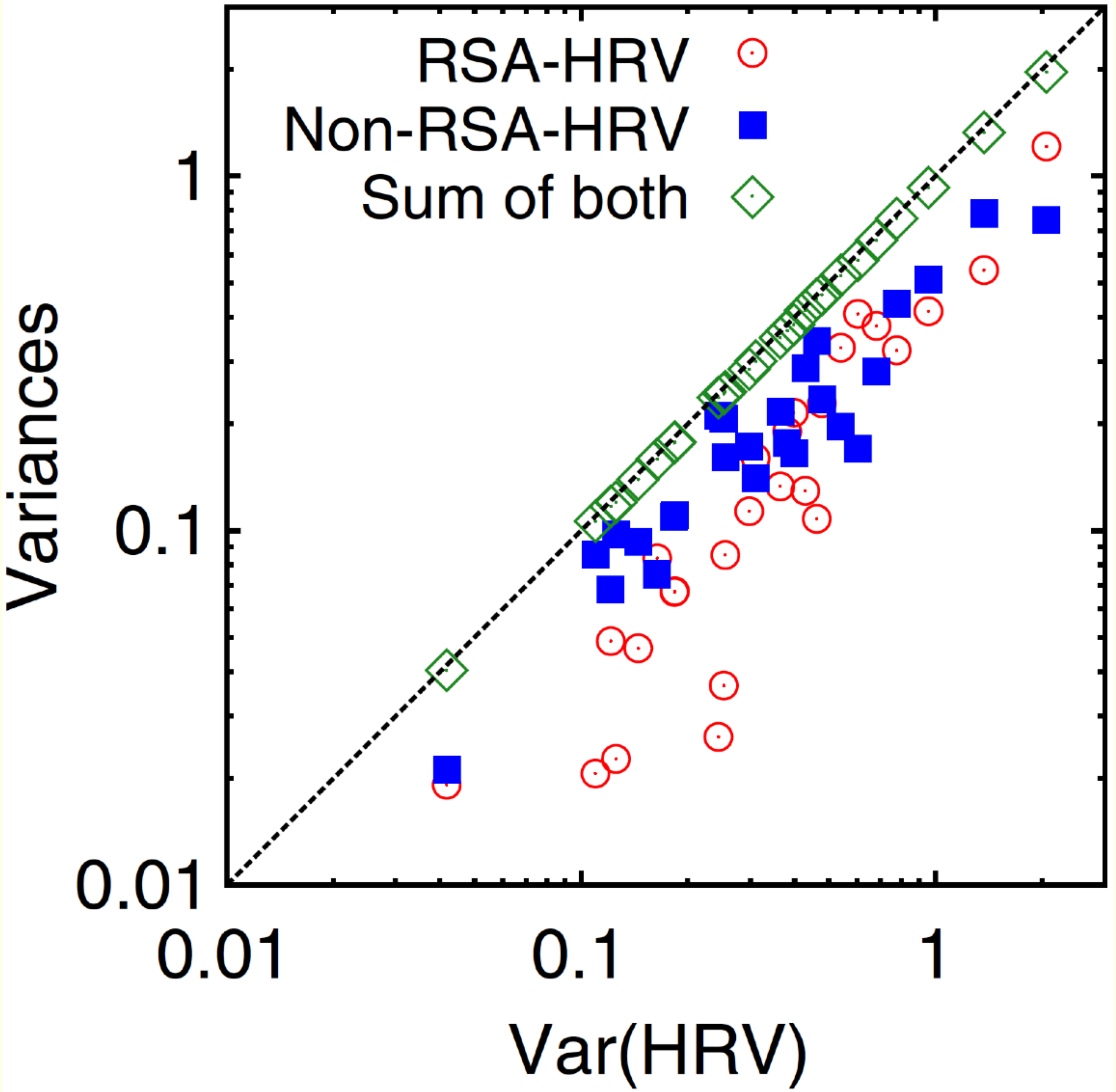
Then we compute time series $\mu_k = Q_R(\varphi_k, \psi_k)$

Then we compute the rest term $\xi_k = \dot{\varphi}_k - \omega - \mu_k$

Thus, $\dot{\varphi}_k - \omega = \mu_k + \xi_k$

HRV=RSA-HRV+non-RSA-HRV

How good is this decomposition?



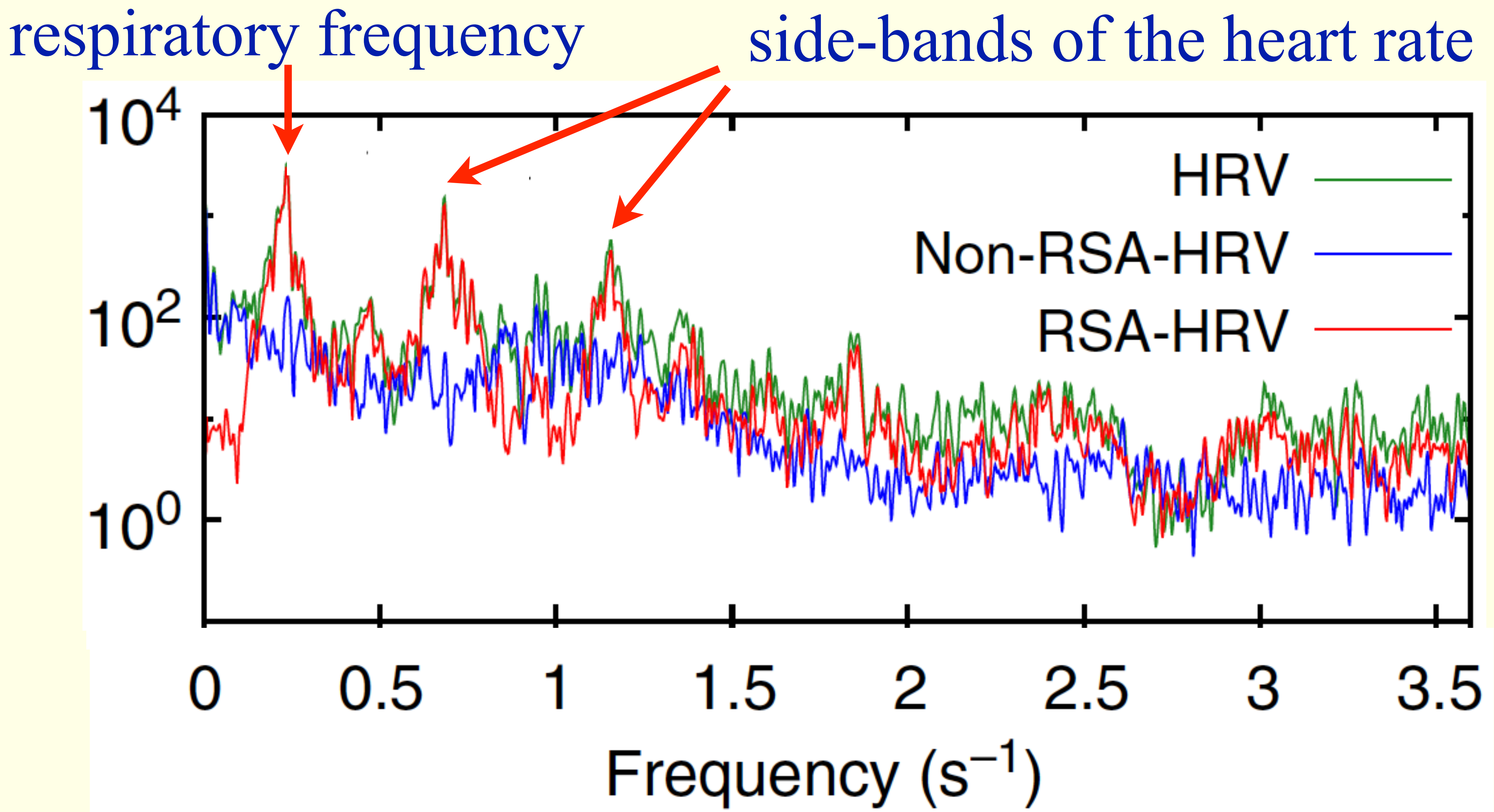
Var(RSA-HRV)+Var(Non-RSA-HRV) $\approx$ Var(HRV)

as expected for non-correlated processes

Decomposition: power spectra

Subject with maximal content of respiratory-related component

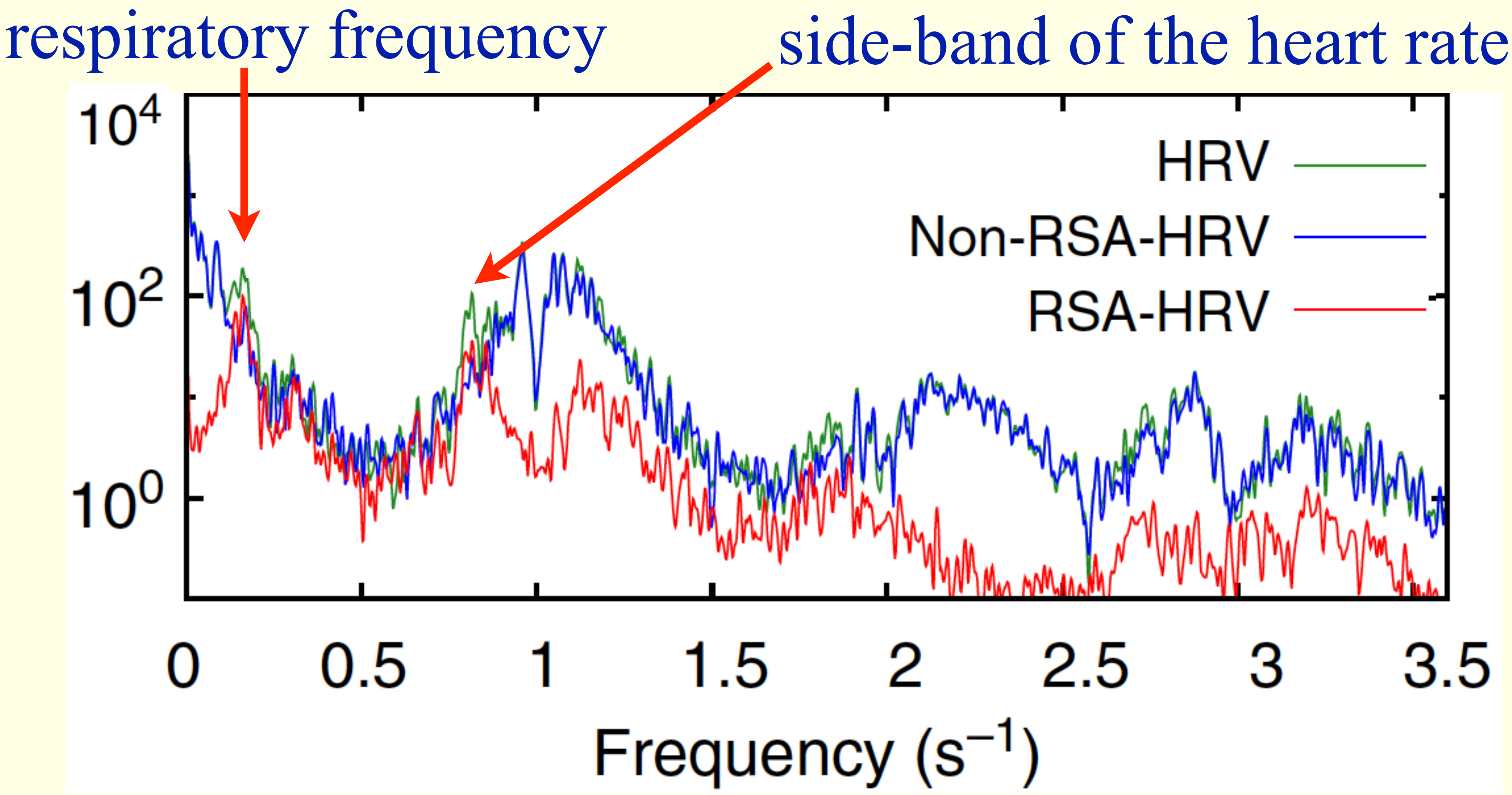
Var(RSA-HRV) $\approx$ 0.67 Var(HRV)



Respiratory-related peaks are well-described by RSA-HRV component

Decomposition: power spectra

Subject with minimal content of respiratory-related component



Even for very weak RSA-HRV component

the respiratory-related peaks are reasonably represented

Summary for this example

Starting with instantaneous phases of cardiac and respiratory systems we disentangled heart rate variability into a component due to respiration and a component due to other factors

However, medical doctors and researchers are used to operate with inter-beat intervals (RR-intervals)

→ We have to generate sequences of RR-intervals for respiratory-related and non-respiratory related components

Phase dynamics with account of many inputs

Cardiac phase from ECG

Phase of respiration

$$\dot{\varphi} = \omega + Q_R(\varphi, \psi) + \xi$$

other inputs and intrinsic fluctuations

Now we introduce two ***new phases:***

φ_R describes effect of respiration (and only respiration!)

and obeys $\dot{\varphi}_R = \omega + Q_R(\varphi_R, \psi)$

φ_{NR} describes effect of everything else except respiration

and obeys $\dot{\varphi}_{NR} = \omega + \xi$

We obtain new phases solving the corresponding equations

(Euler technique)

New RR-intervals

We obtain φ_R, φ_{NR} simulating the corresponding equation

We obtain instants of **respiratory-related R-peaks** from the condition

$$\varphi_R(t_k^R) = 2\pi k$$

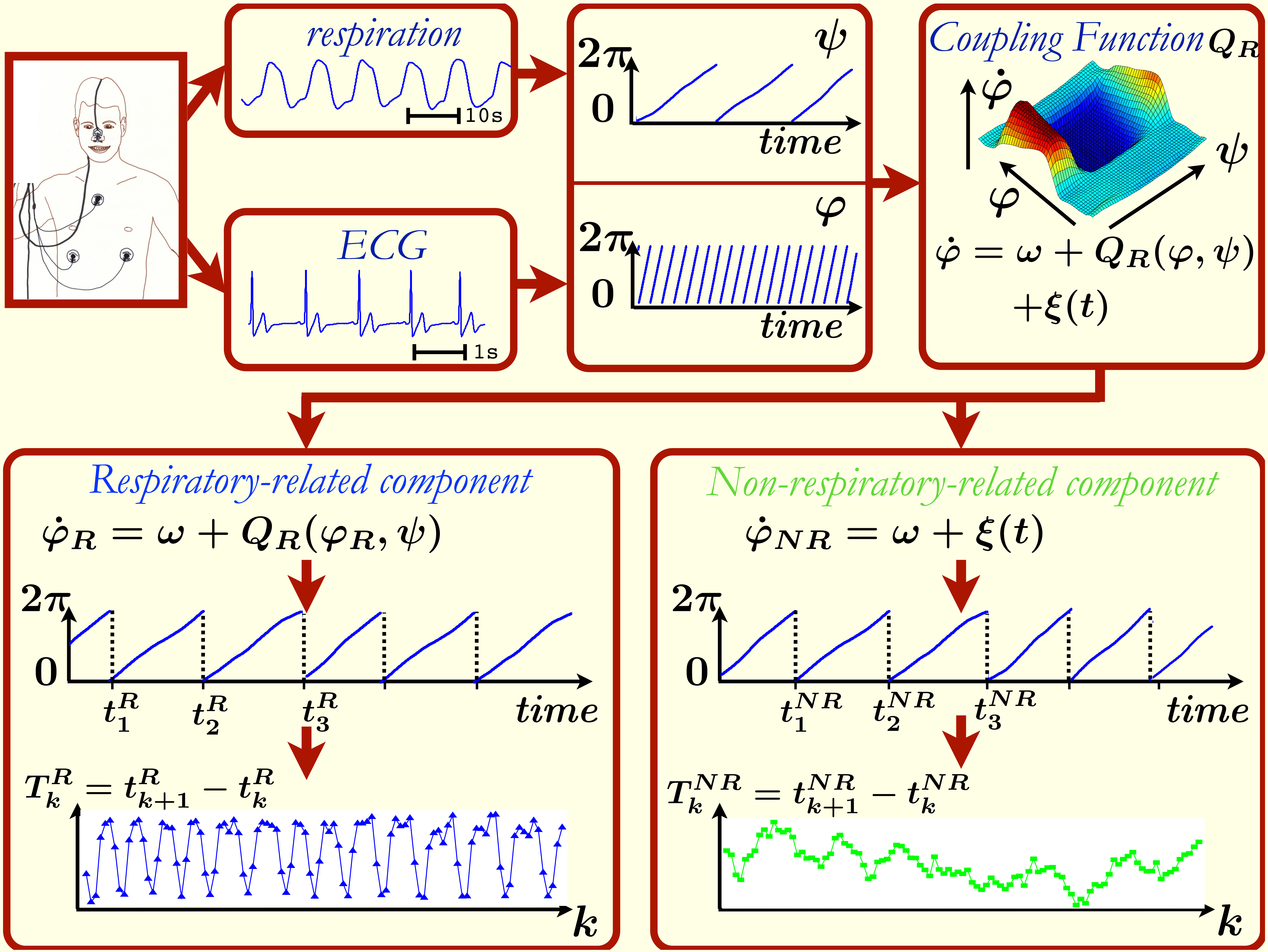
We obtain instants of **non-respiratory-related R-peaks** from the condition

$$\varphi_{NR}(t_k^{NR}) = 2\pi k$$

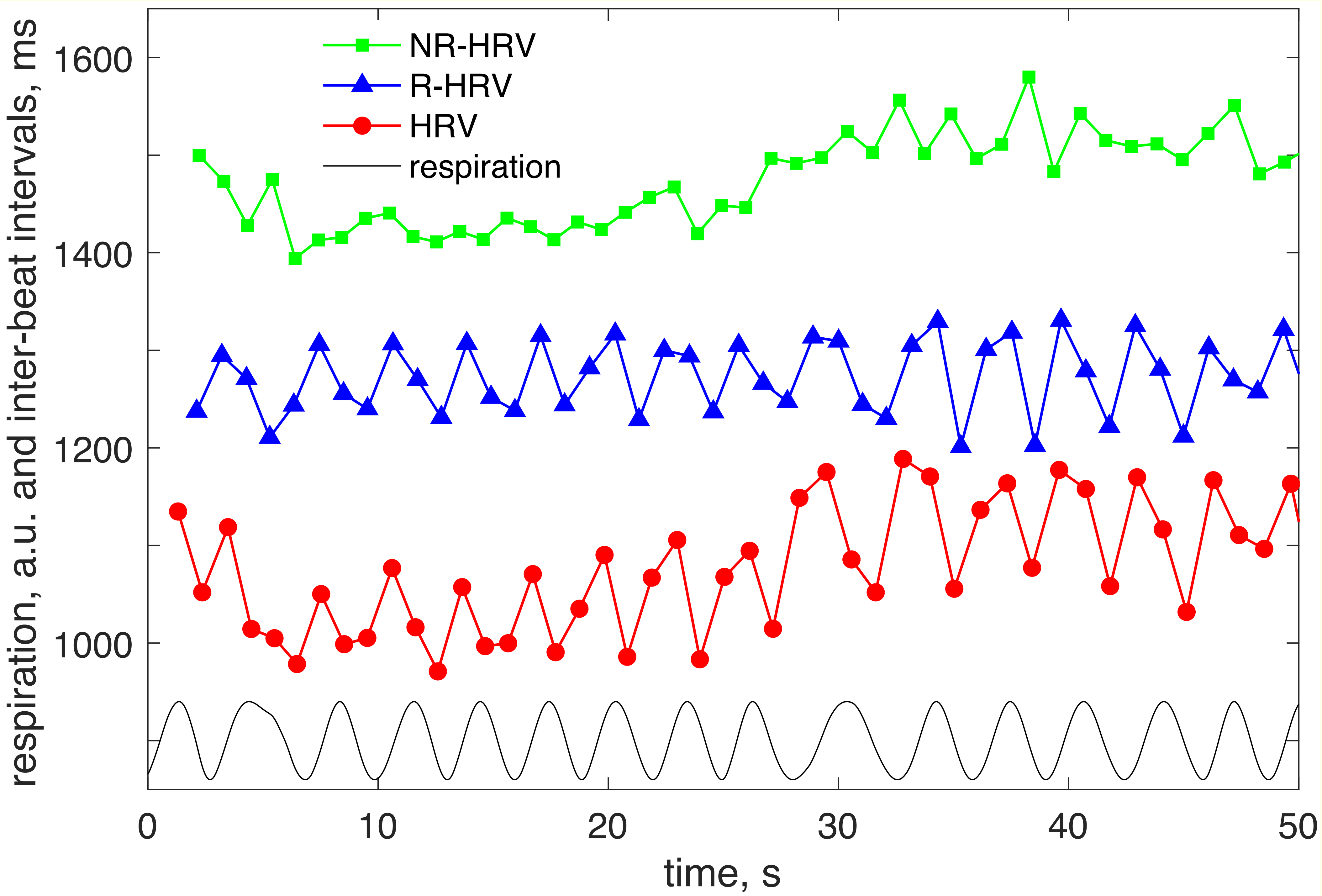
RR-intervals $t_{k+1}^R - t_k^R$: respiratory-related component of HRV

RR-intervals $t_{k+1}^{NR} - t_k^{NR}$: variability due to all other factors

Approach at a glance



An example


NR-HRV
R-HRV
HRV
respiration
respiration, a.u. and inter-beat intervals, ms
1600
1400
1200
1000
0
10
20
30
40
50
time, s

We suggest to use dynamical disentanglement as a universal preprocessing tool prior to computation of any measures of respiratory sinus arrhythmia (RSA)

We computed different time-domain, frequency-domain and complexity measures from RR-series T_k.

Here we show the results for:

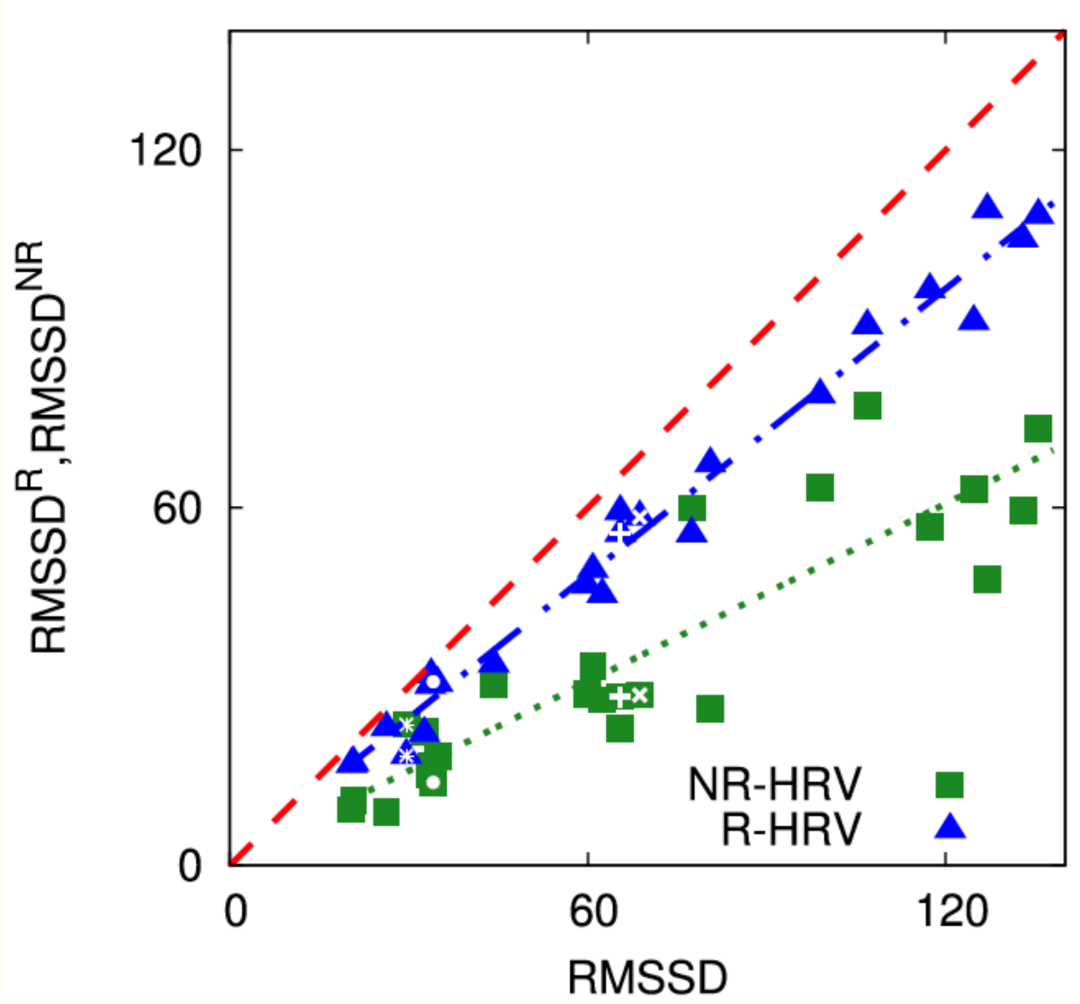
Root mean square of successive differences (RMSSD) (Malik 1996)

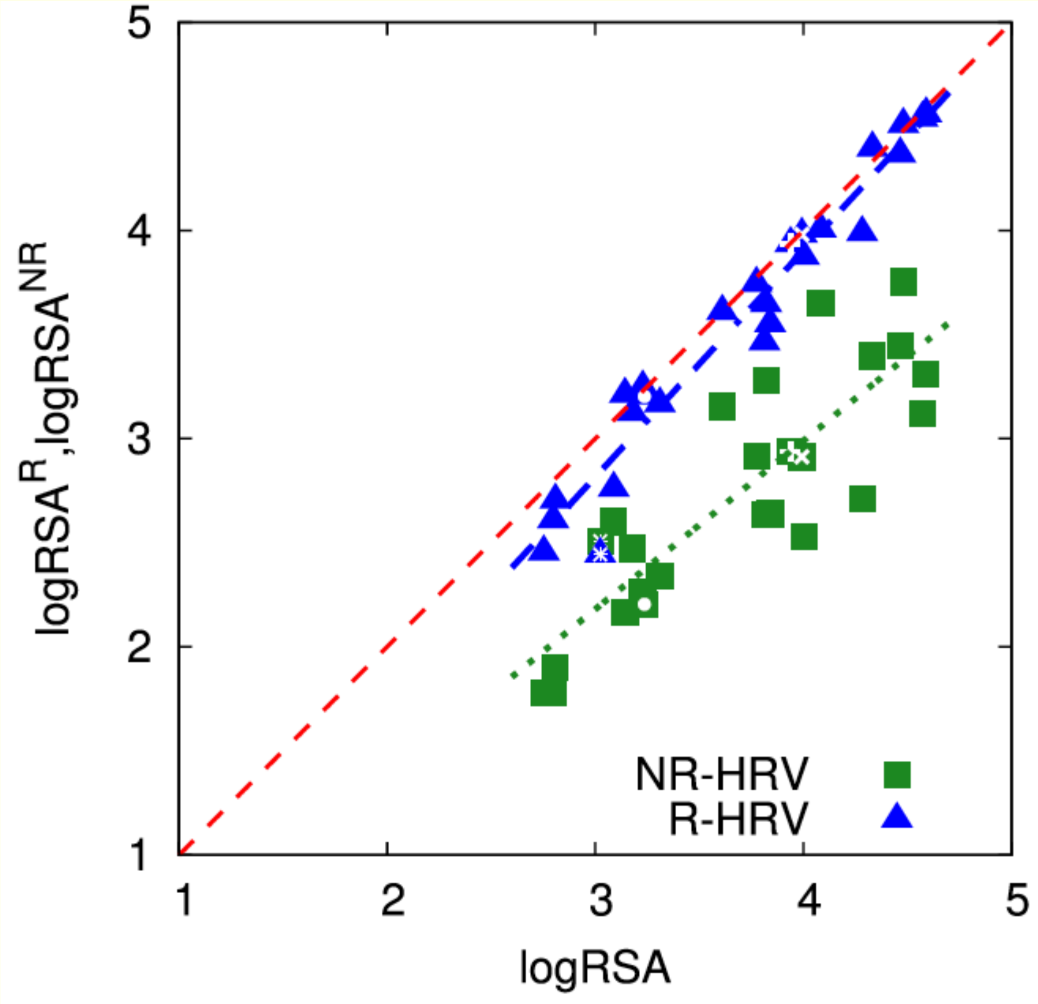
$$\mathrm{RMSSD} = \sqrt{\langle (T_{k+1} - T_k)^2 \rangle_k}$$

Logarithm of the median of the distribution of the absolute values of successive differences (LogRSA) (Lehofer et al 1997)

$$\mathrm{LogRSA} = \log[\mathrm{median}\,|T_{k+1} - T_k|]$$

Analysis of real data (healthy adults)





Çağdaş Topçu[1,2], Matthias Frühwirth[3], Maximilian Moser[1,3], Michael Rosenblum[2,4,5] and Arkady Pikovsky[2,4]

Physiological Measurement

A practical algorithm

- The proposed technique operates with time-continuous phases of the cardiac and respiratory systems, $\varphi(t), \psi(t)$

- Computation of $\varphi(t)$ is quite complicated: it requires high-quality measurements and extensive preprocessing

- Hence, we need a practical (maybe approximate) algorithm that would operate only with R-peaks, i.e. with a point process

A practical algorithm

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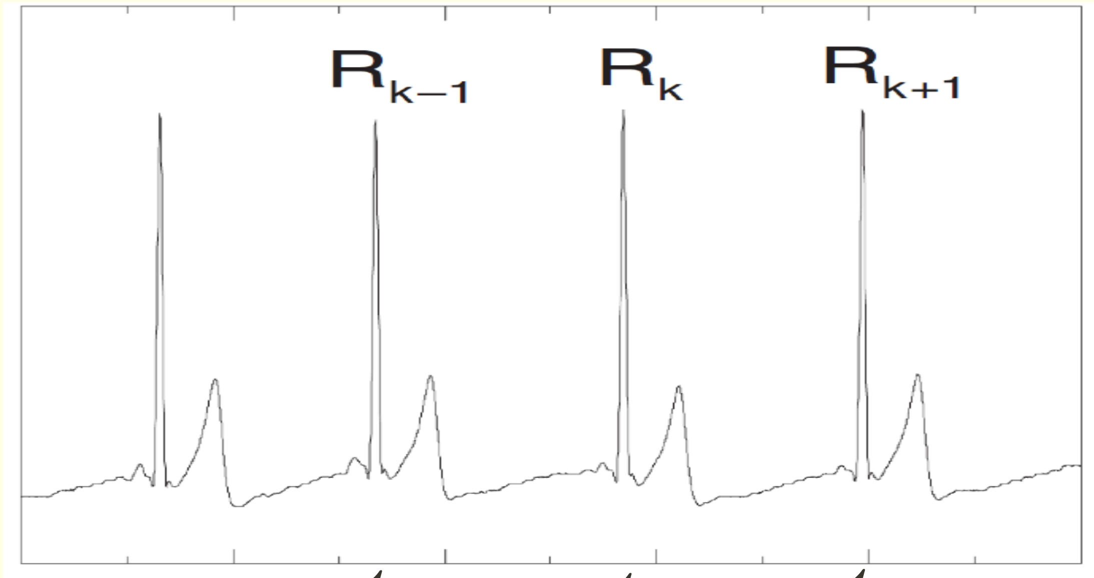
- Computation of $\varphi(t)$ is quite complicated: it requires high-quality measurements and extensive preprocessing
- Hence, we need a practical (maybe approximate) algorithm that would operate only with R-peaks, i.e. with a point process

...and here it is!

Disentanglement from RR-intervals and respiration



t_{k-1} t_k t_{k+1}

Interbeat intervals $T_k = t_{k+1} - t_k$

Recall the equation

$$\dot{\varphi} = \omega + Q_R(\varphi, \psi) + \xi$$

Consider deterministic part and assume weak coupling, $\| Q_R \| \ll \omega$

$$T_k = \int_0^{2\pi} \frac{d\varphi}{\omega + Q_R(\varphi, \psi)} \approx \frac{2\pi}{\omega} - \frac{1}{\omega^2} \int_0^{2\pi} Q_R(\varphi, \psi) d\phi$$

Respiration is much slower than the heart rate

we approximate $\psi(t)$ by a piece-wise linear function:

$$\psi(t) = \psi(t_k) + \omega_k^{(R)}(t - t_k) \quad \text{FOR} \quad t_k \le t \le t_{k+1}$$

Disentanglement from RR-intervals and respiration

$$T_k = \int_0^{2\pi} \frac{d\varphi}{\omega + Q_R(\varphi, \psi)} \approx \frac{2\pi}{\omega} - \frac{1}{\omega^2} \int_0^{2\pi} Q_R(\varphi, \psi) d\phi$$

Respiration is much slower than the heart rate →

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$$\psi(t) = \psi(t_k) + \omega_k^{(R)}(t - t_k) \quad \text{FOR} \quad t_k \le t \le t_{k+1}$$

Then

$$\int_0^{2\pi} Q_R(\varphi, \psi) d\varphi = \int_0^{T_k} Q_R[\varphi(t), \psi(t)] dt \approx F(\psi_k, \omega_k^{(R)})$$

$$T_k \approx T - F(\psi_k, \omega_k^{(R)})/\omega^2 \quad \text{WITH} \quad T = 2\pi/\omega$$

Disentanglement from RR-intervals and respiration

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$$T_k \approx T - F(\psi_k,\omega_k^{(R)})/\omega^2 \quad \text{WITH} \quad T = 2\pi/\omega$$

We introduce mean respiratory frequency $\bar{\omega}$

and represent F as a Taylor-Fourier series:

$$T_k \approx T + \sum_{n=1}^{N_F} \left\{ \left[\sum_{m=0}^{N_T-1} a_{n,m}(\omega_k^{(R)} - \bar{\omega})^m \right] \cos(n\psi_k) + \left[\sum_{m=0}^{N_T-1} b_{n,m}(\omega_k^{(R)} - \bar{\omega})^m \right] \sin(n\psi_k) \right\}$$

N_T, N_F : orders of the Taylor-Fourier series

Disentanglement from RR-intervals and respiration

$$T_k \approx T + \sum_{n=1}^{N_F} \left\{ \left[\sum_{m=0}^{N_T-1} a_{n,m}(\omega_k^{(R)} - \bar{\omega})^m \right] \cos(n\psi_k) + \left[\sum_{m=0}^{N_T-1} b_{n,m}(\omega_k^{(R)} - \bar{\omega})^m \right] \sin(n\psi_k) \right\}$$

Coefficients $a_{n,m}, b_{n,m}$ can be found, e.g., by LMS fit

→ We obtain a **coupling map** for RR-intervals

$$T_k \approx T + \mathscr{F}\left[\psi(t_k), \dot{\psi}(t_k)\right]$$

We take $\omega_k^{(R)} = \dot{\psi}(t_k)$

Construction of the respiratory-related RR-series

→ We obtain a **coupling map** for RR-intervals

$$T_k \approx T + \mathscr{F}\left[\psi(t_k), \dot{\psi}(t_k)\right]$$

Now we construct the respiratory-related RR-intervals:

We take $t_1^{(R)} = t_1$

Substituting $\psi(t_1), \dot{\psi}(t_1)$ into the model we obtain T_1 and

$t_2^{(R)} = t_1^{(R)} + T_1$ … and so on, to obtain all $t_k^{(R)}$

and intervals $T_k^{(R)} = t_{k+1}^{(R)} - t_k^{(R)}$

Construction of the non-respiratory-related RR-series

→ We obtain a **coupling map** for RR-intervals

$$T_k \approx T + \mathscr{F}\left[\psi(t_k), \dot{\psi}(t_k)\right]$$

First, for all original intervals we obtain the rest term (effective noise)

$$\xi_k = T_k - T + \mathscr{F}\left[\psi(t_k), \dot{\psi}(t_k)\right]$$

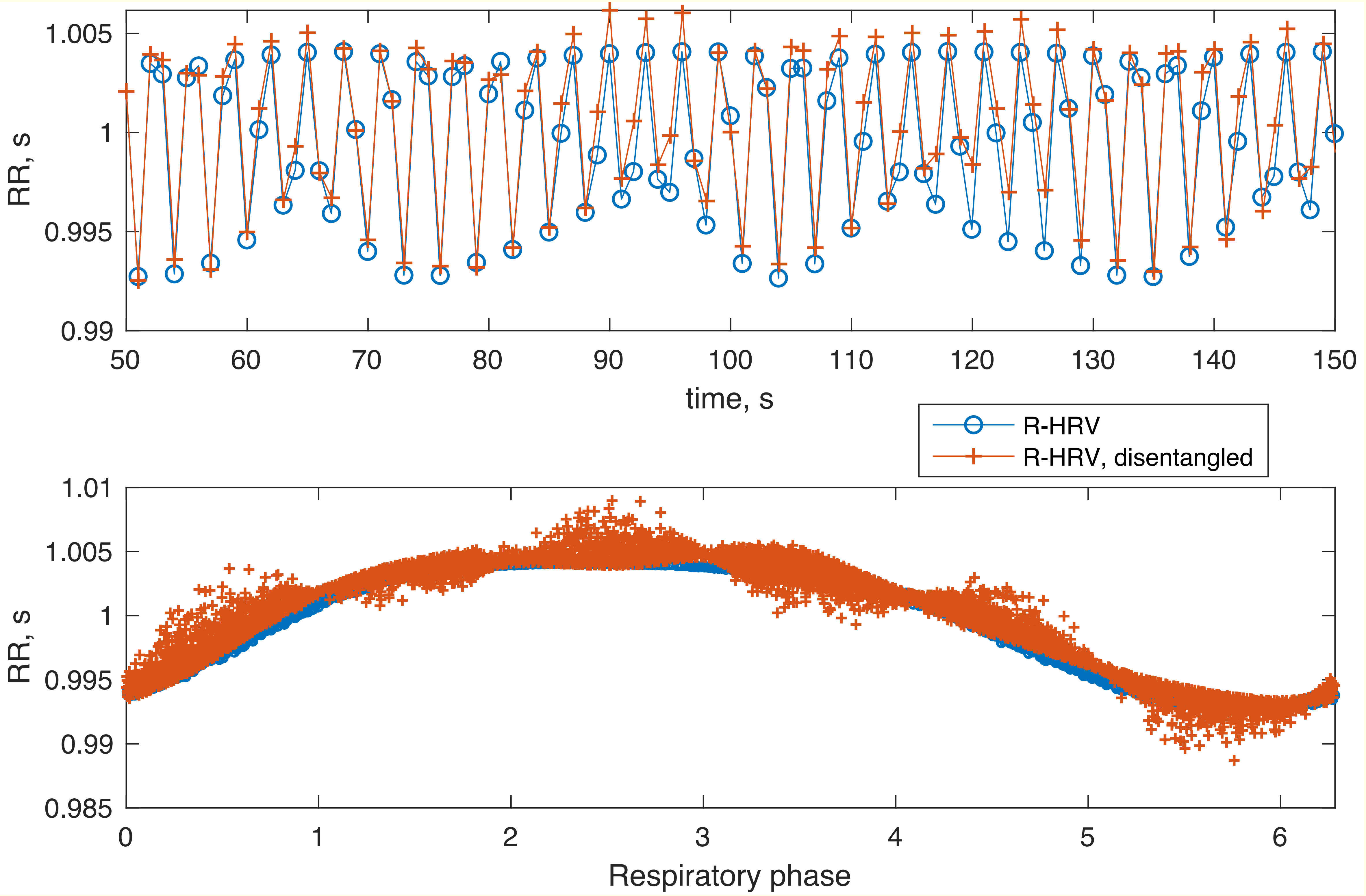
We start with $t_1^{(NR)} = t_1$ and obtain $t_2^{(NR)} = t_1^{(NR)} + T + \xi_1$

Next, if already computed $t_l^{(NR)}$ obeys $t_k < t_l^{(NR)} < t_{k+1}$

then $$t_{l+1}^{(NR)} = t_l^{(NR)} + T + \xi_k + \frac{\xi_{k+1} - \xi_k}{t_{k+1} - t_k}(t_l^{(NR)} - t_k)$$

and $$T_k^{(NR)} = t_{k+1}^{(NR)} - t_k^{(NR)}$$

Results: model data



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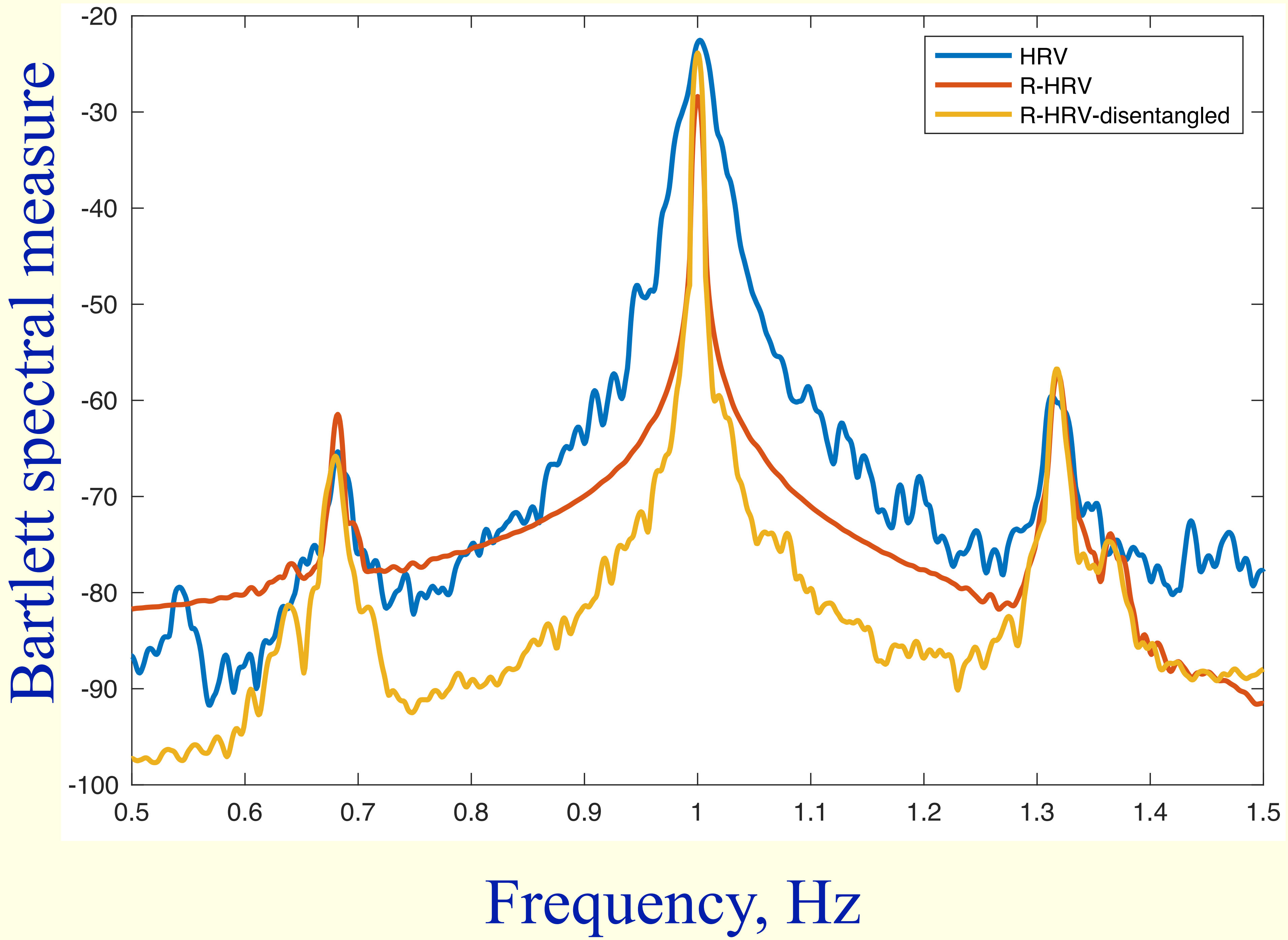
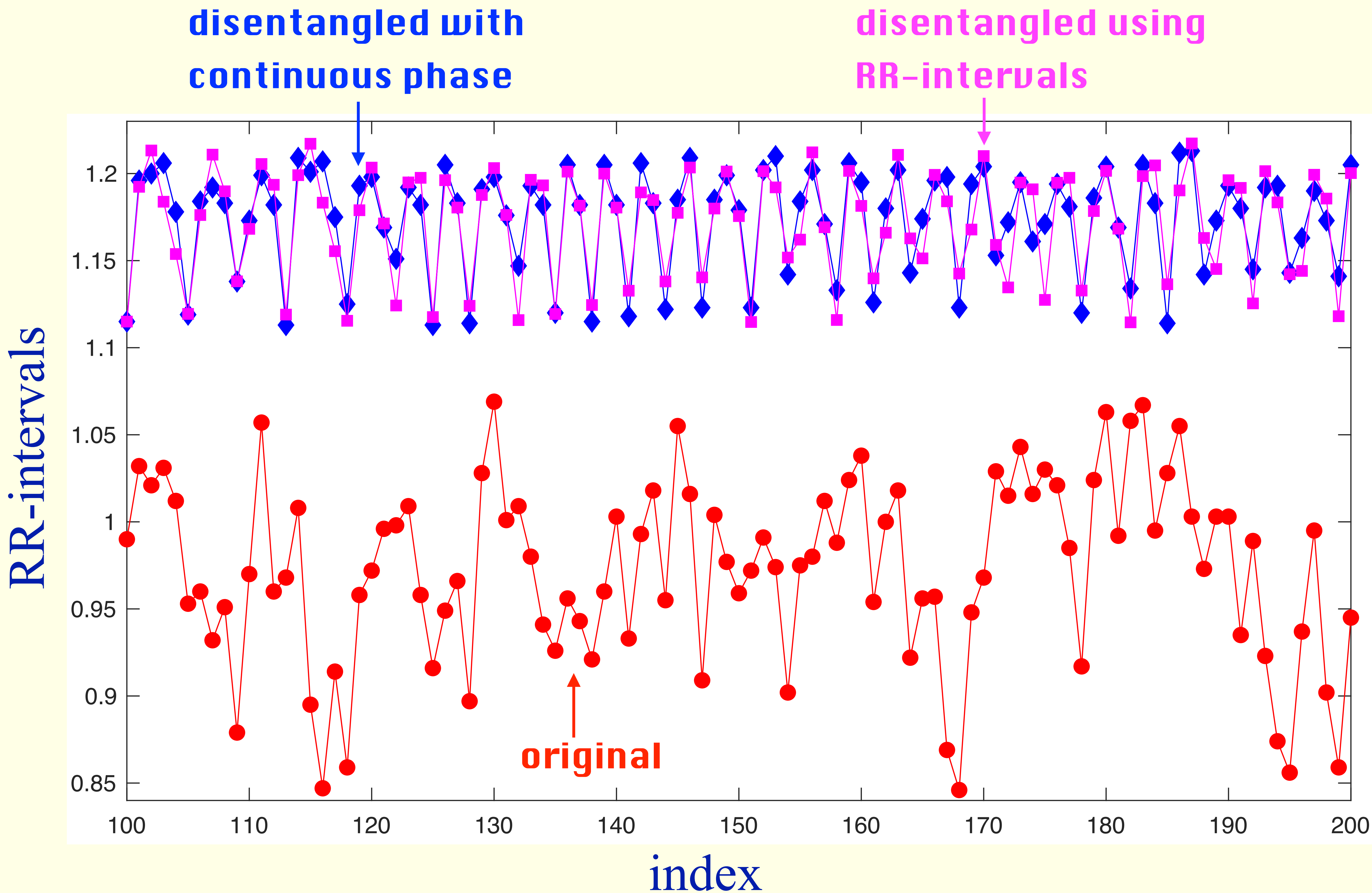


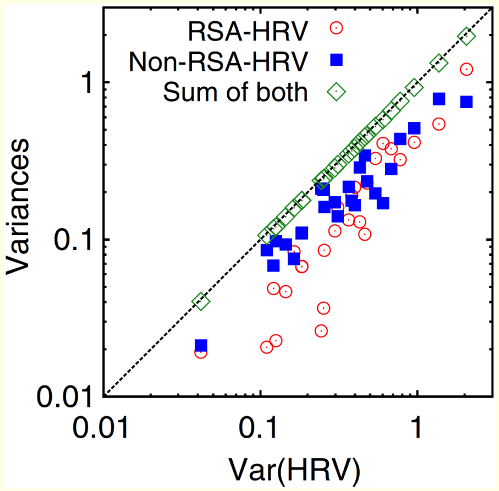
Results: real data



Real data: quality of disentanglement

For continuous phase data we have checked that

Var(RSA-HRV)+Var(Non-RSA-HRV) ≈ Var(HRV)
as expected for non-correlated processes



We now check it for point-process time series of R-peaks, taking the phase to be piece-wise linear between the events,

$$\dot{\varphi}(t) = 2\pi/T_k \text{ FOR } t_k \le t < t_{k+1}$$

and obtaining

$$\sigma^2 = \text{VAR}(\dot{\varphi}(t)) = \frac{4\pi^2}{T_\Sigma} \sum_{k=1}^{N} \left(\frac{1}{T_k} - \frac{N}{T_\Sigma} \right)^2 T_k, \; T_\Sigma = \sum_k T_k$$

We compute variance for 4 series of intervals:

Real data: quality of disentanglement

For continuous phase d

Var(RSA-HRV)+Va
as expected for non

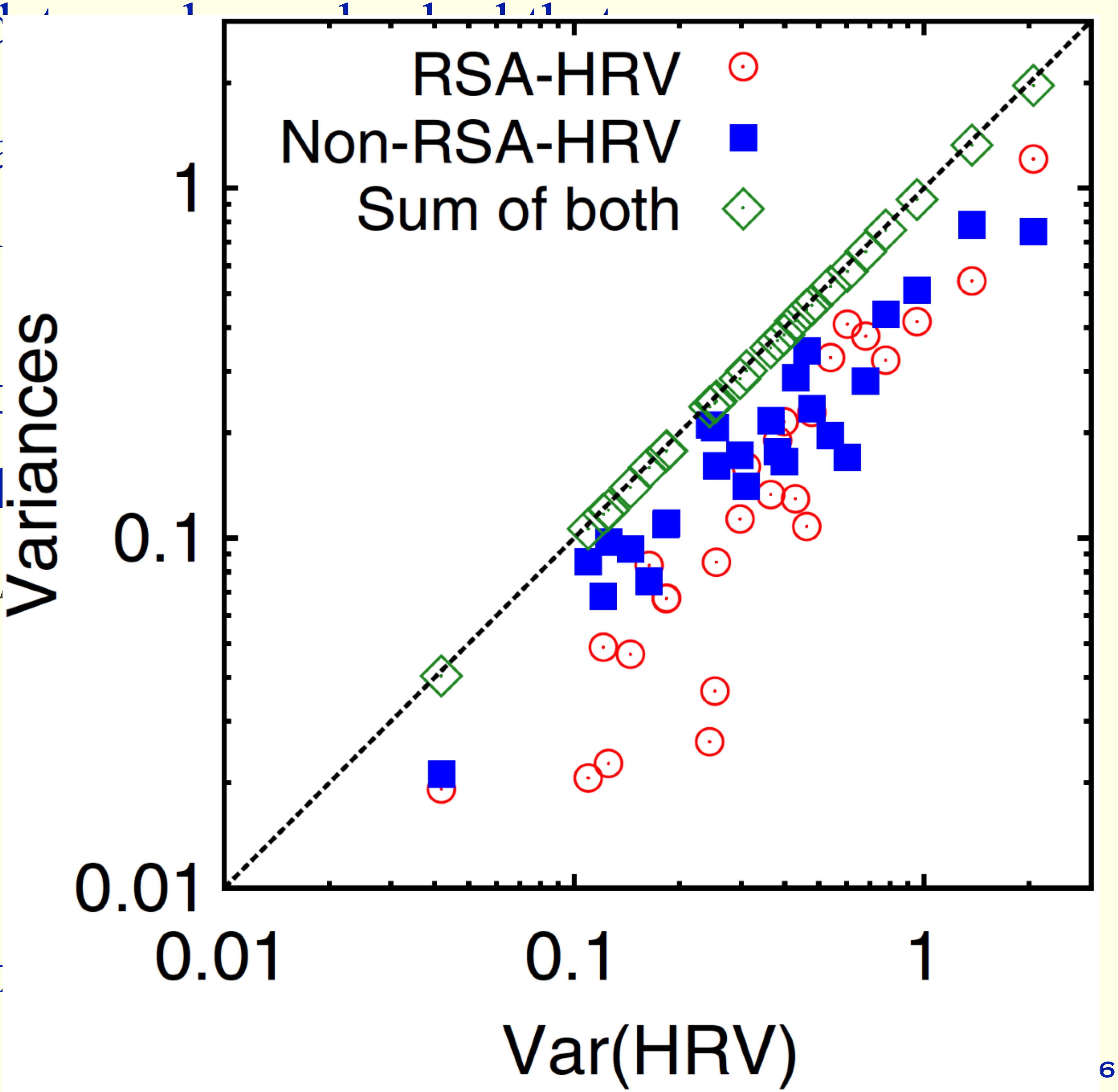
We now check it for po
taking the phase to be p

$$\dot{\varphi}(t) = 2\pi$$

and obtaining

$$\sigma^2 = \text{VAR}(\dot{\varphi}(t)) =$$

We compute variance



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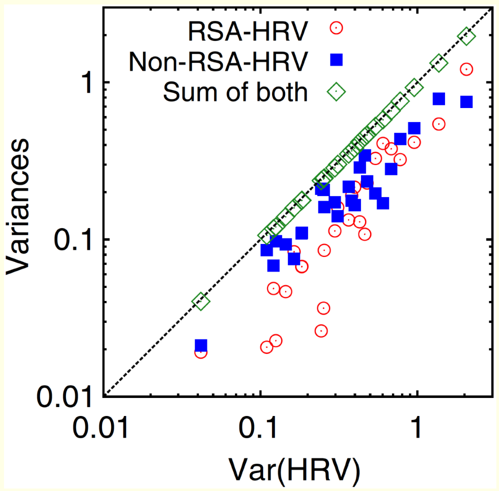

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Real data: quality of disentanglement

$$\dot{\varphi}(t) = 2\pi/T_k \text{ FOR } t_k \le t < t_{k+1} \qquad \sigma^2 = \text{VAR}(\dot{\varphi}(t))$$

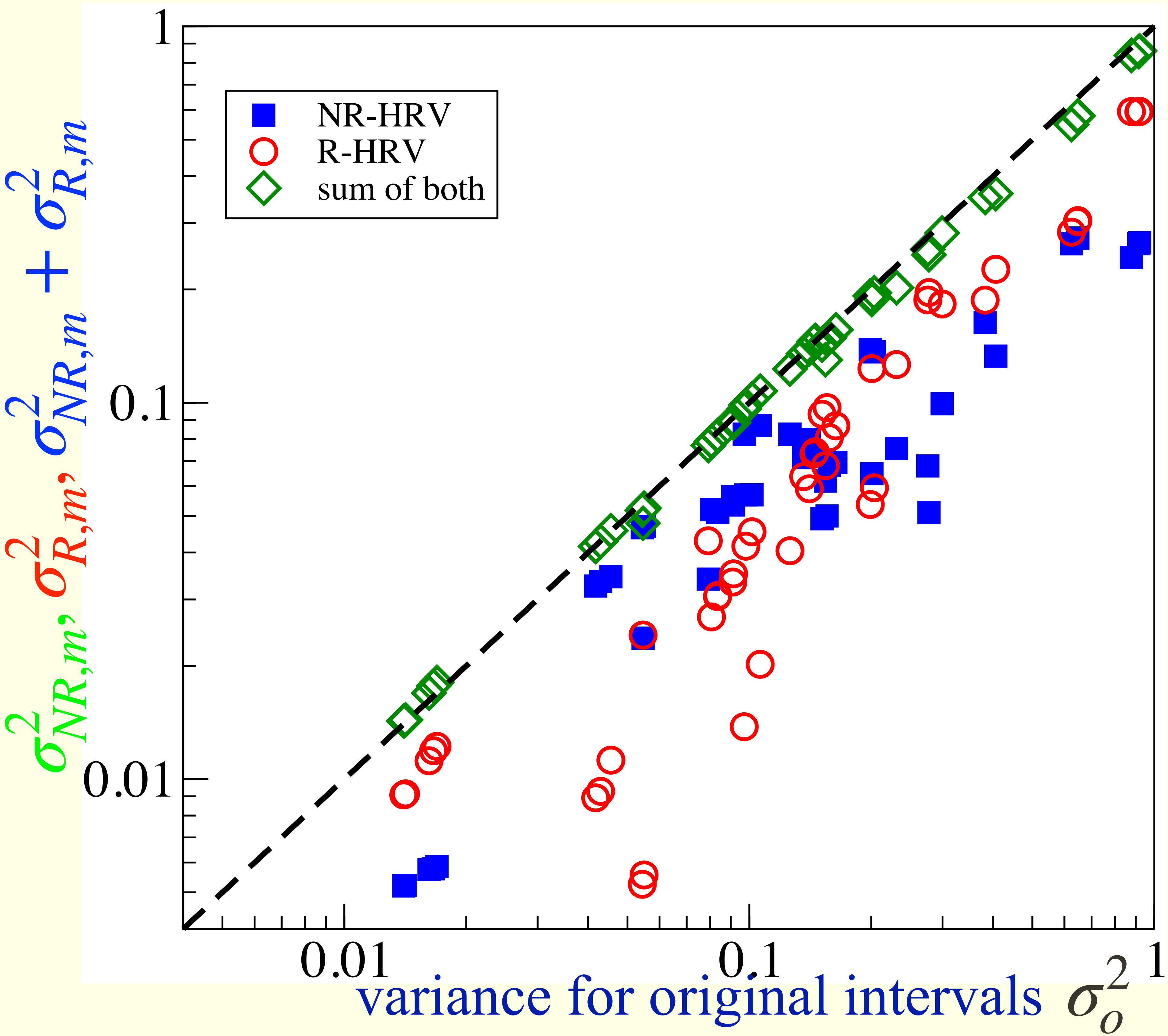
We compute variance for 4 series of intervals:

- variance σ_o^2 for original intervals
- variance $\sigma_{R,c}^2$ for **continuously-cleansed** respiratory-related intervals,
- variance $\sigma_{R,m}^2$ for **map-cleansed** respiratory-related intervals
- variance $\sigma_{NR,m}^2$ for **map-cleansed** non-respiratory-related intervals

We check that: $\sigma_{R,c}^2 \approx \sigma_{R,m}^2$

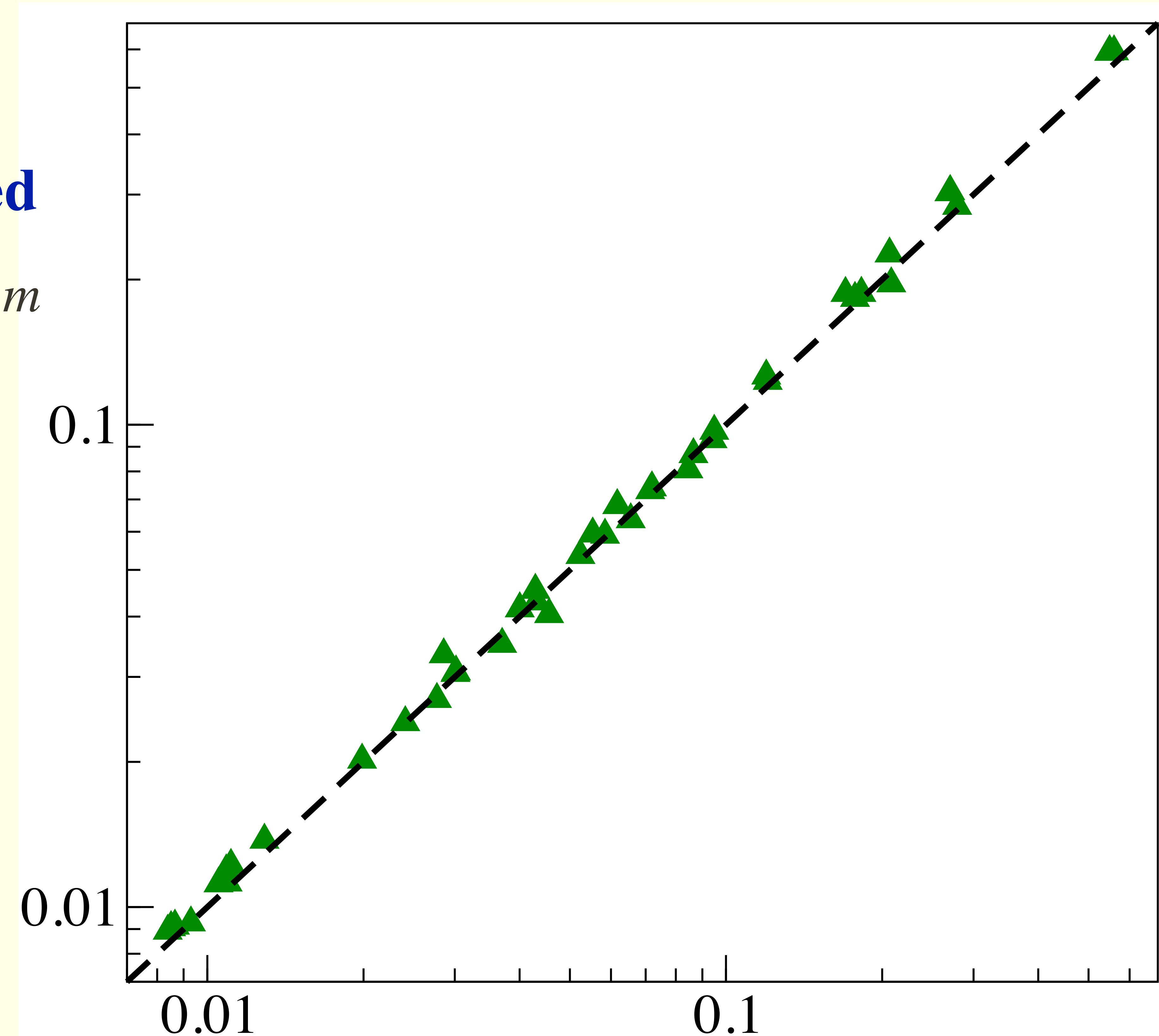
$$\sigma_{R,m}^2 + \sigma_{NR,m}^2 \approx \sigma_o^2$$

Real data: quality of disentanglement



Real data: quality of disentanglement

variance for **map-cleansed** intervals, $\sigma^2_{R,m}$



variance for **continuously-cleansed** intervals, $\sigma^2_{R,c}$

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